Chapter 6

Other Topics / Refinements

Missing Judgments

The pairwise comparison matrices each contain redundant judgments in the sense that there are N x (N-1)/2 judgments in a matrix of size N, when in fact, only N-1 judgments are required to solve for priorities using simple algebra. The redundancy is very useful, however, as it improves accuracy in a manner somewhat analogous to estimating a quantity by taking the average of repeated (and hence redundant) observations. The increased accuracy permits priorities to be calculated even for less accurate or fuzzy judgments, such as when words are used instead of numbers.

An additional property related to the redundancy of judgments is the measure of consistency. Saaty[1] defined a measure of deviation from consistency, called a consistency index, as:

$$C.I. = (\lambda_{max} - N)/(N-1)$$

Saaty calculated a consistency ratio (C.R.) as the ratio of the C.I. to a random index (R.I.) which is the average C.I. of sets of judgments (from a 1 to 9 scale) for randomly generated reciprocal matrices. Harker[2] discussed three reasons why one would want to make fewer than the full set of N(N-1) judgments for each of one or more sets of factors in an AHP model:

- to reduce the time to make the judgments
- because of an unwillingness to make a direct comparison between two particular elements
- because of being unsure about some comparisons

Harker also proposed a method of dealing with missing entries. This method, which is also very useful in helping to reduce inconsistency has been programmed into Expert Choice. If one identifies a judgment possibly in

---

[1] T. L. Saaty, The Analytic Hierarchy Process, McGraw-Hill, New York (1980).

[2] P. T. Harker, "Alternative Modes of Questioning in the Analytic Hierarchy Process", Math Modeling, Vol. 9, No. 3-5, pp. 353-360, 1987, Pergamon Journals.

error, either by manual inspection or with the help of a computerized algorithm, then it is useful to be able to treat the judgment in question as missing, and calculate what it should be in order to be as consistent as possible with the other judgments.

Using Hard Data

Priorities can be derived from data as well as from pairwise comparisons – provided a linear or inverse linear relationship is deemed to be reasonable. For example, the relative preference for a set of alternatives with respect to longevity can be derived directly from expected time to failure data for each of the alternatives. Assuming a linear relationship, an alternative with an expected time to failure of 4 years would be twice as preferable as one with an expected time to failure of 2 years. Simple arithmetic (usually carried out by a computer) is adequate to derive the priorities by adding up the expected time to failures, and dividing by the total to normalize such that the priorities add up to one. The relative preferences for three alternatives with time to failure of 2, 4 and 6 years would be approximately .167, .333 and .500 respectively. Inverse relationships, appropriate when a higher data value is less desirable, can be calculated in a similar fashion. For example, the relative preference with respect to cost for three alternatives costing $2, $4, and $5 respectively would be .545, .273, .182 since the first is twice as preferable as the second and three times as preferable as the third.

The temptation to derive priorities from hard data is difficult to resist. For one thing, the data might have been costly and time consuming to gather, so why contaminate it with human judgment? Secondly, an evaluation that uses as much 'hard data' as possible gives the appearance of being more 'objective'. But it is often advisable to resist deriving priorities directly from hard data, and use the hard data as the basis and substantiation for judgments instead. Preferences are often not linearly related to data. A vehicle with a top speed of 240 mph is not twice as preferable as a vehicle with a top speed of 120 mph for ordinary drivers. A vehicle with a top speed of 240 mph might, however, be twice as preferable as one with a top speed of 200 mph for a racing car driver. The human judgment that is required to translate the data into ratio scale priorities (using pairwise comparisons) adds to the accuracy of the evaluation – provided the human is knowledgeable! This judgment can be applied using verbal, numerical or graphical comparisons.

Converting to Pairwise

It is sometimes desirable to convert data to pairwise ratios and then adjust the ratios using human judgment. Expert Choice has a command to convert data to pairwise comparisons after which the pairwise comparisons can then be modified to reflect any non-linearities in utility appropriate to the evaluation.

Transformation of data

AHP is not intended to replace calculators or spreadsheet programs but to help synthesize the results from many analyses and perspectives. Financial calculations, in particular, are best made with a spreadsheet program, the results of which can be applied in an AHP model. Year by year income and expenses can be entered, totaled, and discounted if desired, to produce a net present value for each alternative under consideration. The financial objective might be evaluated only in terms of net present value, but more typically, other objectives such as initial investment, financing arrangements, and cash flow are important as well.

Although financial calculations are not normally done in an AHP model, Expert Choice does have a feature to transform financial data (or other hard data) to produce results that are comparable to what one would do in a spreadsheet. Consider the following example involving two financial objectives – interest income and capital appreciation:

Table 1 – Interest and Capital Appreciation

Alternative	**Interest**	**Capital Appreciation**	**Total**
Alternative A	$120	$1500	$1620
Alternative B	$160	$1200	$1360
Alternative C	$250	$ 900	$1150
Total	$530	$3600	$4130

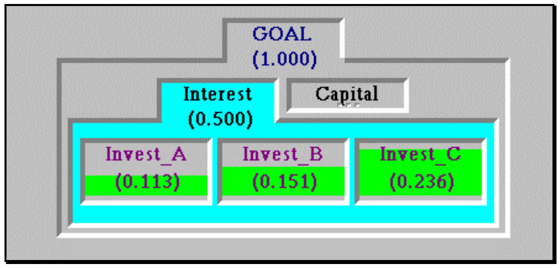


Figure 1 – Priorities with respect to Interest Income

Figure 1 and Figure 4 show the priorities of the alternatives with respect to Interest and Capital Appreciation based on the data entered directly into an Expert Choice model. The priorities for the objectives, Interest and Capital, should not, however, be equal, since capital appreciation is so much more than interest income.

Rather than making pairwise comparisons as to the relative importance of interest and capital appreciation, the priorities can be derived from the data entered for the alternatives. The priorities[3], shown in, Figure 2 are precisely the ratio of $530/$3600. A synthesis of this small model (which in

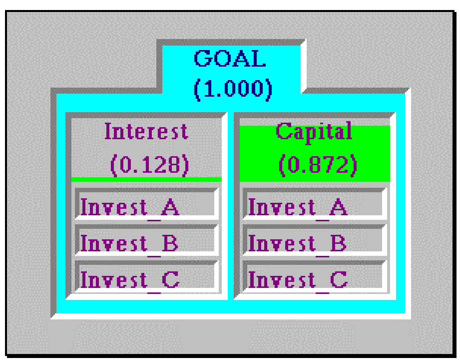


Figure 2 – Data transformed to calculate priorities of objectives

[3] From the Expert Choice Transformation option at the goal to determine the priorities of the children based on use the data in the grandchildren.

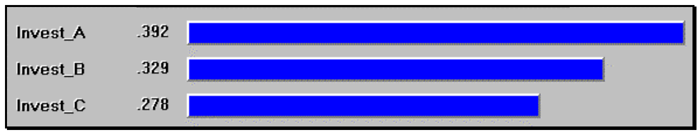


Figure 3 – Alternative priorities

reality would be part of a larger model) results in the alternative priorities shown in Figure 3. The ratio of these priorities is the same as the ratio of the total incomes of the three alternatives: $1620: $1360: $1150.

Artificial Clustering of Elements – Linking Clusters

Earlier we talked about how the decomposition principle of AHP is applied to structure a complex problem into a hierarchy of clusters, sub-clusters, sub-sub clusters and so on. The main reason for clustering elements is to better cope with complexity. A second reason is to reduce the number of pairwise comparisons required. Pairwise comparison of 24 nodes would require 24 x 23 / 2, or 276 comparisons. Pairwise comparing three clusters of 8 nodes each would require 3 x 8x 7/2 +3 (to compare the three clusters) = 87 comparisons in all. Still another reason to organize elements into subclusters is accommodate situations where the disparity between elements in a group is so great that they are not of the same "order of magnitude". We also discussed why each cluster should have no more than seven, plus or minus two elements.

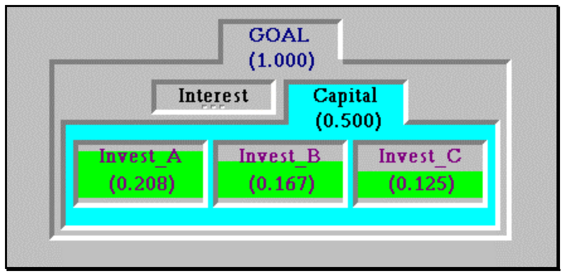


Figure 4 – Priorities with respect to Capital Appreciation

Occasionally, a decision-maker may have more than nine nodes in a cluster and will not want to, or cannot cluster them into meaningful sub-clusters.

It is possible to organize the elements into artificial clusters of up to nine elements each and perform pairwise comparisons of the elements within each of the clusters. When deriving the priorities for the artificial clusters, the Expert Choice "link elements" option will select the first element from each of the artificial clusters for pairwise comparisons. The priorities for the artificial clusters that result from these pairwise comparisons are then calculated such that the ratios of the *first elements* within each of the clusters correspond to the pairwise comparisons of these elements.

Ratings Approach

Absolute vs. Relative Measurement

The two ways to prioritize alternatives are known as relative measurement and absolute measurement. When you create a model using all *relative measurement* the priorities of the objectives, sub-objectives *and alternatives* are computed by comparing the elements one to another. In contrast, an absolute measurement model is used to gauge the *alternatives* against an established scale and not against each other. In everyday life we often use absolute measurement – for example, when measuring distance (e.g. miles), volume (e.g. liters), and temperature (e.g. degrees Celsius). In Expert Choice we can derive new scales even where none exist. For example we can establish categories (which we call intensities) into which letters of recommendation for students fall: Excellent, Above Average, Average, Below Average, and Poor; and then prioritize them. After prioritizing the categories, they become the standards or scale for measuring letters of recommendation. Absolute measurement is performed in the Expert Choice Ratings spreadsheet. In a Ratings approach the objectives are pairwise compared against one another as usual, but the alternatives are evaluated using a pre-established scale instead of compared to one another. While some scales such as miles or liters are well established and widely recognized[4], you will often create your own scales that can be customized for your particular evaluation.

The ratings approach combines the power of the hierarchy and the pairwise comparison process with the capability to rate hundreds or thousands of alternatives. Ratings models are used in a wide variety of applications such as college admission decisions, personnel evaluation, and resource allocation (see page 235). The ratings approach looks more like the traditional weights and scores approach for evaluating alternatives in that alternatives, arranged in rows, are rated against the lowest level sub-objectives of the hierarchy, arranged in columns. The priorities of the columns are derived with pairwise comparisons, but pairwise comparisons are *not* used for evaluating the alternatives. Instead, each alternative is rated in each column (representing a lowest level sub-objective) on a scale of intensities specific to that column (lowest level sub-objective).

The hierarchy for a RATINGS model differs from that of a traditional AHP model in that intensities appear at the lowest level of the hierarchy instead of the alternatives.[5] The intensities, in a sense, serve as surrogates for the alternatives. Instead of deriving priorities for the alternatives with respect to each of the lowest level sub-objectives, we derive priorities for the intensities through a pairwise comparison process. For example, instead of

---

[4] Even when working with well defined scales it may desirable to derive a new scale based on the defined scale in order to reflect preference which is often non linear and sometimes even non-monotonic.

[5] A traditional AHP model can be converted to a RATINGS model by deleting the leaves of the Goal plex and then adding intensities below each of the lowest level sub-objectives.

comparing alternatives such as John and Sue with respect to quality of work, we define rating intensities for rating quality of work, such as OUTSTANDING, EXCELLENT, AVERAGE, BELOW AVERAGE, POOR, and make judgments such as: how much more preferable would an employee who produces outstanding quality of work be to one who produces excellent quality of work. Subsequently, the alternatives are entered in as rows of the ratings spreadsheet and rated using the intensity scales specific to each lowest level sub-objective (or column of the spreadsheet).

Do not necessarily reserve the Ratings approach only for models with a large number of alternatives. Sometimes a Ratings model is used even when you have only a few alternatives because it looks more like a traditional evaluation methodology and therefore may be more familiar to those using it, or because there may be rules or regulations that prohibit comparing one alternative against another[6].

An Overview of a Ratings Model

We illustrate the RATINGS approach using an employee evaluation model. Rating intensities must be established for each lowest level sub-objectives. For example, the lowest level sub-objectives in the employee evaluation model are Education, Years of Experience, Quantity of Work, and Quality of Work. We decided that the Education intensity should have the four intensities: High School degree, Bachelor's degree, Masters degree, and Ph.D. There are five intensities for the Years of Experience objective: 1 to 2 years, 3 to 5 years, 5 to 10 years, 10 to 15 years and more than 15 years. The objectives Quantity of Work and Quality of Work have the same intensities: Excellent, Above Average, Average, Below Average, and Poor.

The intensity nodes are inserted into the model under the objectives (or sub-objectives) to which they apply. In the pairwise comparison process you will be asking questions such as: "How much more preferable a worker who is Excellent rather than Above Average with respect to Quality of Work?" When the priorities have been determined, you, the decision

[6] Government procurement rules sometimes prohibit comparing one alternative against another.

maker, have derived a new customized scale, a standard with which to consistently measure each alternative with respect to the overall decision goal, based on your experience. Some scales may be widely used by others, while some will be specially derived scales created to measure some objectives or criteria for which no scale previously existed.

Creating the Ratings Model from Evaluation and Choice

Like the relative measurement models, Ratings models can be created directly in EVALUATION AND CHOICE. However, instead of alternatives, you will enter intensity scales at the bottom of the hierarchy. The alternatives are then entered into the Ratings spreadsheet for the final comparisons.

The main steps for creating a Ratings model are:

- Open a new file
- Edit Insert:
    - Goal
    - Objectives
    - Sub-objectives - from one to several levels of sub-objective
    - Intensities
- (Assessment) Pairwise compare all (sub)objectives throughout the model to establish priorities
- (Assessment) Pairwise compare the intensities under the lowest level of (sub)objectives
- (Assessment) RATINGS and enter the alternatives in the RATINGS spreadsheet
- Rate the alternatives in the RATINGS spreadsheet.

A typical ratings model is shown below. The EVALUATION AND CHOICE part of the model includes the goal, objectives, (sub)objectives and has intensities as the leaves of the model.

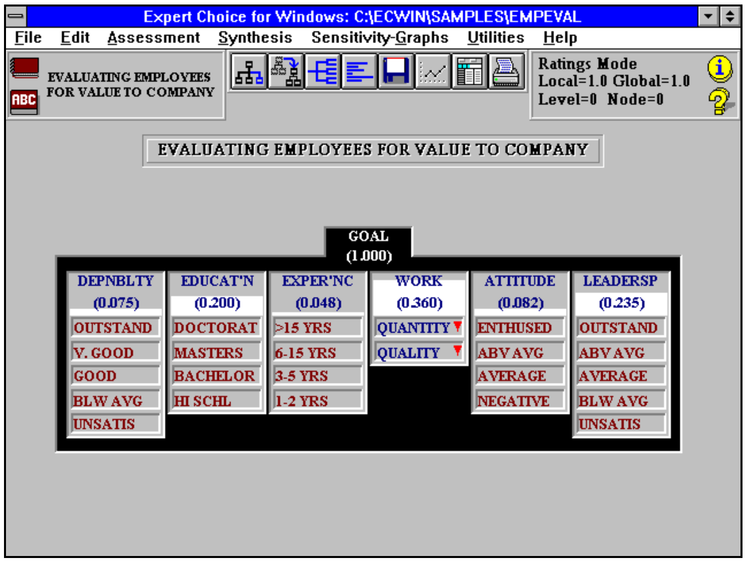


Figure 5 – The EVALUATION AND CHOICE Part of a Ratings Model.

Using Ranges for Intensities

Often people are more comfortable making judgments if the intensity nodes are expressed as a range of values. For example, under the objective, Years of Experience in the employee evaluation model there are four intensity nodes: 1-2 YRS, 3-5 YRS, 6-15 YRS, and > 15 YRS (more than 15 years). Note that the ranges do not have to be equal in span.

In constructing intensity scales the intention should be to convey the information about the alternative being rated and to use ranges that are logical to the evaluator. Years of Experience conveys something about how useful the employee is to the company and an experienced evaluator knows what it means for an employee to have experience of 1-2 YRS or 6-15 YRS. It would perhaps make no sense to the evaluator to distinguish between 9 years and 10 years of experience. Another option would be to name the intensities with category names such as Novice, Some experience, Solid experience, and Senior person.

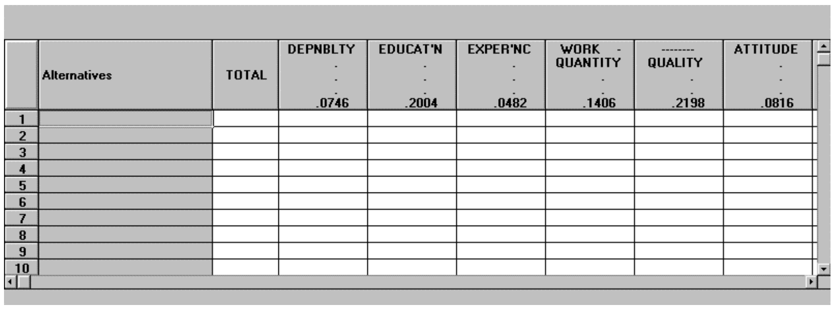


Figure 6 – The Startup Ratings Screen

Using the Same Intensities for all Objectives

Intensities, should be specific to lowest level sub-objective. However a helpful shortcut is to build the first rating set, derive its priorities through the usual pairwise comparison process, and then replicate this set to the leaves, the other bottom level (sub)-objectives in the hierarchy. Then revise the intensities for specific sub-objectives where necessary. For example, an intensity set of Outstanding, Excellent, might be appropriate to the lowest level sub-objectives, but not to all. Furthermore, examine and revise the judgments that are used to derive intensity priorities where necessary. For example, ratio of outstanding to good for appearance of an Air Force Pilot should be different than ratio of outstanding to good for ability to maneuver fighter aircraft in combat.

Note there is a column for each objective or lowest level sub-objective in the ratings spreadsheet. For example, DEPNBLTY, EDUCAT’N, EXPER’NC, and ATTITUDE are objectives and have their own columns. The objective of WORK has two sub-objectives, QUANTITY and QUALITY, so WORK itself does not have a column, but it’s sub-objectives do.

From absolute to relative measurement

After prioritizing the alternatives using the ratings approach it is possible to select a subset of the alternatives (up to nine) and extract them to a temporary E&C model (named $$$) in order to: (a) view sensitivity graphs, and/or (b) refine the priorities with pairwise relative comparisons.

The E&C model named $$$ that is created by extracting up to 9 alternatives from a ratings spreadsheet will, down to the lowest level of sub-objectives, be identical to the E&C model on which the ratings spreadsheet is based. The alternatives that were extracted from the ratings spreadsheet will replace the original intensities under each of the lowest level sub-objectives. Data for the alternatives will based on the absolute ratings entered in the ratings spreadsheet. If one of the alternatives extracted is an 'ideal' alternative (received the highest intensity for every lowest level sub-objective), then an ideal mode synthesis will produce priorities for the alternatives in exactly the same ratio as those obtained in the absolute ratings mode.

Sensitivity graphs can be viewed for the alternatives extracted from the ratings spreadsheet. In addition, the priorities of the alternatives with respect to the lowest level sub-objectives can be 'refined' through the pairwise comparison process, by converting the alternative data to pairwise judgments and adjusting the judgments. For example, two alternatives that performed approximately the same might be each rated as Excellent with respect to one of the objectives then their data values will be the same. When these data values are converted to pairwise, the judgment between the two will be equal. However, a decision maker might, when comparing one directly to the other, feel that one is a bit more preferable, and can adjust the pairwise judgment accordingly. If adjustments are made to pairwise comparisons, the $$$ temporary model should be saved under a permanent name.

Ideal and Distributive Synthesis Modes (Preventing or allowing rank reversals)

Researchers in decision analysis fields that developed prior to AHP, (expected utility theory and multiattribute utility theory) based their work on axioms that did not allow rank reversals to occur when "irrelevant alternatives", are introduced for consideration. More specifically, the axiomatic base of utility theory includes an assumption like the one made by Luce and Raiffa[7]:

> "Adding new acts (alternatives) to a decision problem under uncertainty, each of which is weakly dominated (preferred) by or is equivalent to some old act, has no effect on the optimality or non-optimality of an old act."

The above axiom is strengthened into the axiom that states the principle of the *independence of irrelevant alternatives*, Luce and Raiffa (1957, p288):

> "If an act is non-optimal for a decision problem under uncertainty, it cannot be made optimal by adding new acts to the problem."

In contrast, AHP can either allow or prevent rank reversals. AHP, as originally conceived by Saaty (1980), assumes what we will call a "closed" system in allocating priorities to alternatives. By a closed system we mean that the sum of the priority *distributed* to the alternatives from each lowest level (sub)criterion does not increase or decrease if new alternatives are added or existing alternatives are removed from consideration. Multi-attribute Utility Theory (MAUT) on the other hand, employs what we will refer to as an "open" system because the sum of the priorities allocated to alternatives will increase or decrease as new alternatives are added or existing alternatives are removed. As will be illustrated below, the ranking of alternatives in a "closed" system can change when a new but dominated (or so called irrelevant) alternative is added to a decision. We will refer to a changing of rank when an irrelevant alternative is added to the decision as a rank reversal.

---

[7] Luce, R.D. and H. Raiffa, 1957, *Games and Decisions*. John Wiley and Sons, Inc., New York., p. 288

Discussion of rank reversal in what follows will always be in the context of the introduction or removal of an irrelevant alternative as there is no debate about the reversal of rank when a relevant alternative, i.e., one that is not dominated on every criterion is added to or removed from a decision. Rank reversal cannot happen in an "open" system. A debate has been ongoing between practitioners of AHP and MAUT about whether the rank of alternatives should be *allowed* to change when an "irrelevant" alternative is added to the decision. AHP practitioners have argued that a change in rank is legitimate. MAUT practitioners have argued that it is not. Each side has presented examples where their argument has appeal. As we will see, both sides are correct, but under different circumstances. We will see that when dealing with a "closed" system, rank adjustment is not only legitimate, but is often desirable. Conversely, when dealing with an "open" system, rank adjustment should be precluded. Rank adjustment, or what MAUT practitioners call rank reversal, occurs when the ranking of a set of alternatives changes upon the introduction of a so called "irrelevant alternative" – an alternative that is dominated by one or more previously existing alternatives. There are two basic misconceptions about this phenomenon. First, the description of such an alternative as "irrelevant" is misleading. Huber, Payne, and Puto[8] (1982) state that "the very presence of [a] dominated alternative results in quite different choice probabilities among the remaining alternatives than in the pristine state, where such items are never considered." This is certainly true when using AHP relative measurement as any alternative is a fortiori relevant since all other alternatives are evaluated in terms of it. We illustrate this with an example on page 156.

The second misconception is about the effect that irrelevant alternatives "should" have in an evaluation. Some MAUT practitioners demand that "irrelevant" alternatives "should" not affect the ranking of other alternatives. This is sometimes referred to as an "independence of irrelevant alternative" assumption. For example, Dyer[9] (1990) cautions about generating "rank

[8] Huber, J., Payne, J. W., and Puto, C., 1982. Adding Asymmetrically Dominated Alternatives: Violations of Regularity and the Similarity Hypothesis. Journal of Consumer Research. Vol. 9, June, 90-98.

[9] Dyer, J. S. 1990. Remarks on The Analytic Hierarchy Process, Management Science, Vol. 36, No. 3, 249-258.

orderings that are not meaningful with respect to the underlying preferences of the decision maker" when additional alternative(s) are introduced for consideration. Although it is possible for any algorithm to generate ranks that do not agree with the underlying preferences of a decision maker, there is nothing to have us believe that a rank adjustment is necessarily contrary to the underlying preferences of decision makers. Furthermore, the arbitrary prohibition of rank adjustment may *lead* to flawed results because there are many situations where a rank adjustment (reversal) is desirable. This will be illustrated below.

The Cause of Rank Adjustment

Rank reversal does *not* occur because of eigenvector calculations, because of the nine point scale used in AHP, because of inconsistencies in judgments, nor because "exact" copies are included in an evaluation. Forman (1987) gives an example where there is perfect consistency and where the introduction of new alternatives causes a rank reversal. The example given by Belton and Gear (1982) has an "exact" copy, but a similar example by Dyer (1990) does not. Dyer argues that "The defense of the AHP on the grounds that copies should not be allowed as alternatives is without foundation, and cannot be supported on intuitive or on technical grounds." We agree in part — copies should be allowed. However, the "defense" of AHP, or its strength, is that it can adjust rank when copies are introduced and this in fact can be supported on both intuitive and technical grounds as will be shown later.

Rank reversal can take place with *any* technique that decomposes and synthesizes in a relative fashion, regardless of whether it uses pairwise comparisons, eigenvector calculations, or demands perfect consistency. There is agreement between AHP and MAUT practitioners. Rank reversal occurs because of an *abundance* or *dilution* effect (or what has also been called a *substitution* effect). This is discussed in Saaty[10] (1990), Dyer[11]

[10] Saaty, T. L. 1990. An Exposition of the AHP in Reply to the Paper "Remarks on the Analytic Hierarchy Process", Management Science, Vol. 36, No. 3, 259-268.
[11] Dyer, J. S. 1990. Remarks on The Analytic Hierarchy Process, *Management Science*, Vol. 36, No. 3, 249-258.

(1990), and Forman[12] (1987), and illustrated below. Since value or worth is, more times than not, affected by relative abundance or scarcity, the ability for a methodology to adjust rank is often a desirable property.

Saaty[13] (1991b), discusses conditions under which one might justifiably say that rank can and should be preserved and when it should not. While some authors (Belton and Gear[14] (1982), Shoner and Wedley[15] (1989), and Dyer (1990)) have suggested that the choice of a modeling approach be based on rank reversal considerations, we propose that a more fundamental and meaningful consideration is whether scarcity is or is not germane to the decision. If scarcity is germane then a closed system (distributive synthesis) is appropriate and rank reversal should be allowed to occur. If scarcity is not germane, then an open system (ideal synthesis) is appropriate and rank reversal should not be allowed to occur. Consequently, a robust decision methodology should be able to accommodate either situation. An extension to AHP to allow modeling both open and closed systems will be presented below. Other modifications to AHP, such as B-G modified AHP (Belton and Gear 1982) and Referenced AHP (Schoner and Wedley 1989) have been advocated in order to prevent rank reversal. While these modifications hold merit and are in fact the same as, or similar to, the open system (ideal synthesis) of AHP discussed below, the merit does not stem from preventing rank reversal nor should these modifications replace the conventional AHP since rank reversals are sometimes, perhaps even often, desirable.

Closed and Open Systems – Scarcity and Abundance

In a "closed" system with a *fixed amount of resources*, scarcity is germane. The distribution of a country's gold, the allocation of a corporation's R&D budget, and the distribution of votes to political candidates are good examples. Suppose a newly formed country decided to

---

[12] Forman, E. H., 1987. Relative Vs. Absolute Worth, *Mathematical Modeling*, Vol. 9, No. 3-5, 195-202.

[13] Saaty, T. L. 1991b. Rank and the Controversy About the Axioms of Utility Theory -- A Comparison of AHP and MAUT, *Proceedings of the 2nd International Symposium of The Analytic Hierarchy Process*, Pittsburgh, PA, 87-111.

[14] Belton, V. and Gear, T. 1982. On a Shortcoming of Saaty's Method of Analytic Hierarchies. *Omega* Vol. 11 No. 3, 226-230.

[15] Schoner, B. S. & Wedley, B. W, 1989, "Ambiguous Criteria Weights in AHP: Consequences and Solutions", Decision Sciences, 20, 462-475.

distribute its gold reserve to identified segments of society based on evaluation objectives that included population and economic potential. Suppose that after distributing the gold a previously overlooked segment of the society with a small population but great economic potential was identified. In order to make a distribution to this segment, gold would have to be taken back from the existing segments and redistributed. Because the previous distribution was made partly on the basis of population and economic potential, sectors that were highly populated but with relatively low economic potential would lose less than segments with relatively low population and high economic potential and a rank reversal could occur. Similarly, a conservative independent candidate entering the race for the U.S. Presidency in which the Republican candidate had the lead might appeal more to Republican voters than to Democratic voters and a rank reversal might take place between the Republican and Democratic candidates.

In contrast, scarcity is not germane in an "open" system where *resources can be added or removed.* As an example, consider the distribution of a new country's currency. Suppose a new country was deciding how to distribute currency to identified segments of society based on evaluation objectives that included population and economic potential. If after distributing its currency, a new segment appears, more currency can be printed and distributed to the new segment based on its population and economic potential (as well as the other evaluation objectives). There is no need to take back currency from existing segments. While the percentage of wealth (in currency) of the previously existing segments would diminish because more currency was printed, the relative amounts of currency and hence the rank order of the segments would not change.

The assumptions of an open system can be better understood by defining a reference "unit of wealth". Suppose there were an alternative that was best on every objective used in the evaluation – an "ideal" alternative.[16] A reference "unit" of wealth is the amount of wealth that this "ideal" alternative would receive. Each real alternative, or segment of society in this

---

[16] This is in contrast with a utopian alternative which could be defined as having the best conceivable values on each objective.

example, would receive some percentage of the reference "unit". Subsequently, the relative wealth of each alternative can be found by normalizing over all real alternatives.

In the currency distribution example, if a new segment of society is introduced and additional currency is printed for the new segment, the ratios and rank order of wealth for previously existing segments will not change.

Closed and Open Synthesis Modes with AHP.

The treatment of closed and open systems leads us to the following two AHP synthesis modes. We next discuss, in turn, Closed and Open Synthesis modes with AHP.

Closed System (Distributive Synthesis)

When priorities are distributed in an AHP hierarchy of objectives and sub-objectives, the global priority of the goal (standardized to 1.0) is distributed to the objectives, and subsequently to the lowest level sub-objectives. (This also would be true for an MAUT hierarchy of objectives). In the original AHP implementation (which we will henceforth refer to as the closed system or distributive synthesis) the priorities of the lowest level sub-objectives are *distributed* to the alternatives in the same fashion. If, for example, an objective's global priority were .4 (see Figure 7) and the local priorities of the three alternatives under the objective were .5, .3 and .2, the global priority of the objective would be distributed to the three alternatives as global priorities of .2, .12, and .08 respectively; see Figure 8.

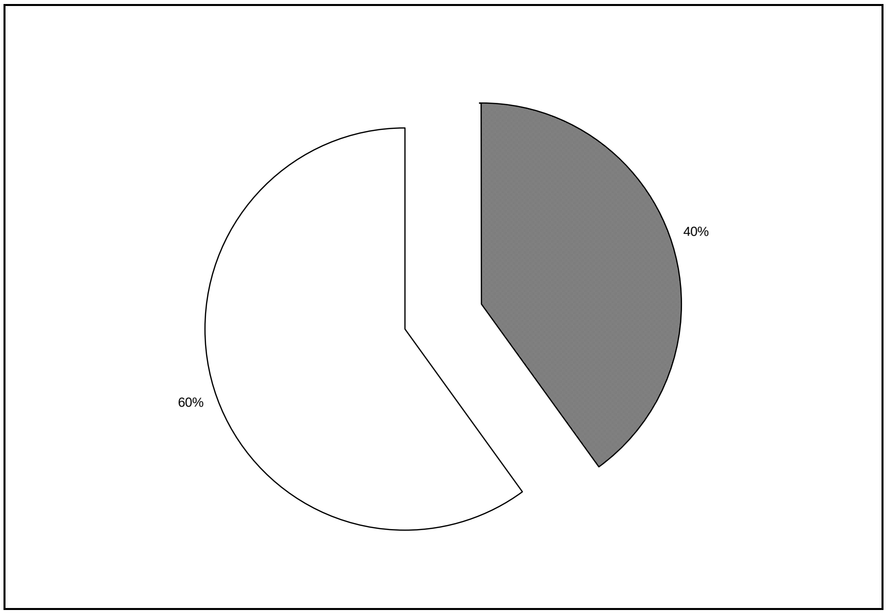


Figure 7 – Evaluation Objective with .4 global priority

If a new alternative were added to (or removed from) the analysis, the existing alternatives would lose (or gain) priority under each objective. For example, if a new alternative D, identical to the second best alternative B, were added under this objective, the local priorities would change to 0.385, 0.231, 0.154, and 0.231. The distribution of the .4 priority of the objective would be 0.154, 0.092, 0.062, and 0.092 as shown in Figure 9. The system is “closed” in that the total priority of the alternatives under each (sub)objective will not change and the total priority for the alternatives under all (sub)objectives will always equal 1.0.

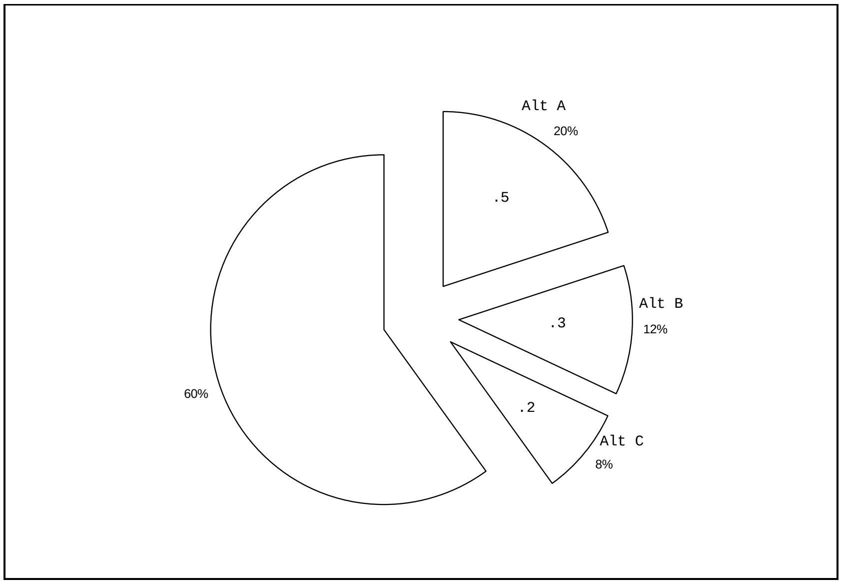


Figure 8 – Evaluation objective priority distributed to the alternatives

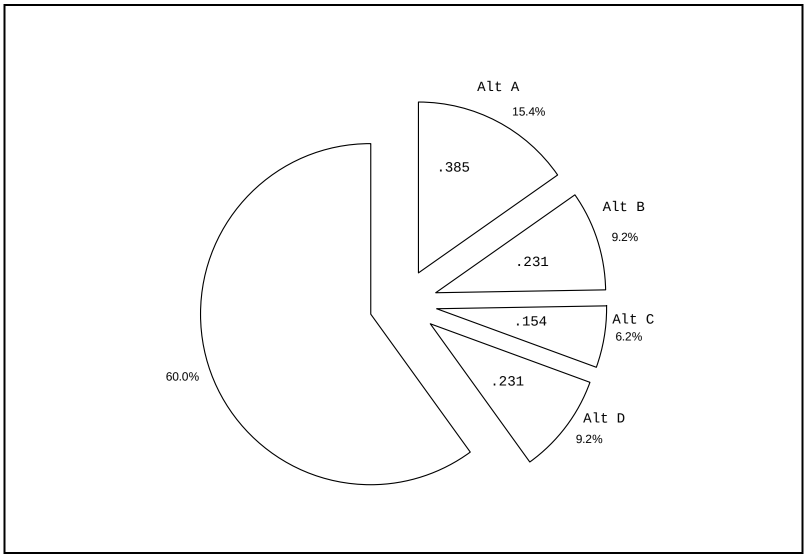


Figure 9 – Distribution after adding a fourth alternative

Open System (Ideal Synthesis)

A simple extension has been made to AHP in order to model open systems. Instead of distributing each (sub)-objective's priority to the alternatives, the priority is allocated to the alternatives such that the most preferred alternative under each (sub)objective receives the full priority of the (sub)objective. This idea was first proposed by Belton and Gear as a replacement for, rather than an extension to, AHP. Each of the other alternatives receives a priority *proportional to its preference relative to the most preferred alternative*. For example, if an objective's priority were .4 (as shown in the right hand segment of the pie chart in Figure 8) and the local priority of three alternatives under the objective were .5, .3 and .2 respectively, the three alternatives would receive global priorities of .4, .24 and .16 respectively as shown in Figure 10.

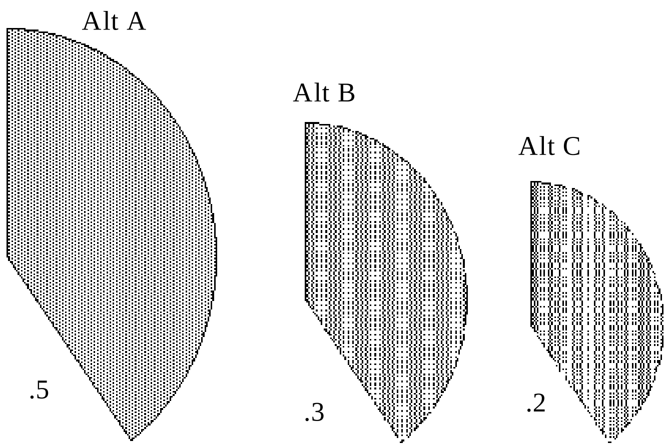


Figure 10 - Three alternatives under the .4 priority objective

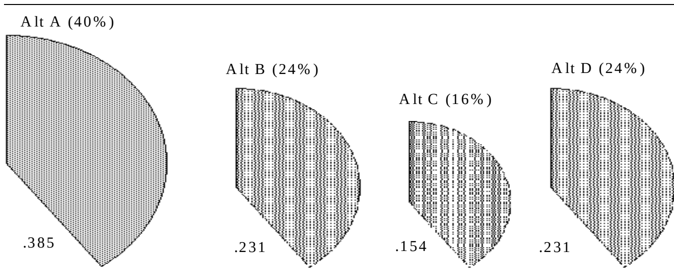


Figure 11 – Ideal mode: Objective with .4 global priority and four alternatives

If, as above, a new alternative that is identical to the second best alternative under this objective were added, the local priorities would again change to 0.385 0.231 0.154 and 0.231. However, the allocation of the .4 priority of the objective using the open system (ideal synthesis) would be .4, .24, .16, and .24 as shown in Figure 11.

The rationale for this approach is that an "ideal" alternative (an alternative having the most preferred attribute value for every objective) would serve as a reference and receive a total priority (before normalization) of 1.0, while each real alternative would have priorities proportionately less. If we think of 1.0 as representing a "standard", each real alternative receives some fraction of the "standard" depending on how well the alternative compares to the ideal on each objective. Although, no alternative can receive a priority from a (sub)-objective greater than the (sub)-objective's priority, the sum of the alternatives' priorities under a (sub)objective is not limited as in the "closed" system. If a new, irrelevant alternative is added (or an existing irrelevant alternative is removed), the priorities allocated to the existing alternatives under each objective do not change because the alternative being added (removed) is, by definition, not better than the ideal under any objective. Therefore, the ideal continues to receive the full priority of the respective objective. Furthermore, the priorities allocated to the other alternatives, being proportional to the ideal, would not change either. However, the alternative being added (removed) would receive (relinquish) a priority in proportion to its preference with respect to the ideal alternative. Thus the total priority allocated increases (decreases) and the system can be said to be "open". After the alternatives receive priority from each of the lowest level (sub)objectives, a subsequent normalization is performed so that the alternative priorities sum to 1.0.

It is important to note that the only operational difference between the open and closed system occurs when a synthesis is performed. There is no difference in model structure or judgment process for closed or open systems. Also of importance is that both closed and open AHP systems produce *ratio scale* priorities. Ratio scale priorities have a higher level of measure than, and are preferred to interval scale priorities. If, in the open system (ideal synthesis), a transformation was made whereby the 'worst' alternative for each objective received 0 priority (in addition to the transformation assigning all of an objective's priority to the 'best' alternative), the mathematics would be analogous to that of MAUT but the resulting priorities would only be on an interval scaled rather than on a ratio scale. Not only are ratio scale priorities more preferable in general, but ratio scale priorities are *required* for many applications (such as resource allocation or systems with feedback) since the product of interval scaled measures is mathematically meaningless.

Illustrative Example

Consider the evaluation and ranking of employees in a small firm with a few employees. Suppose that Susan is as good or slightly better than John with respect all attributes except one – John is the only employee who is proficient in application of personal computers in meeting the needs of the firm's clients. Suppose a multi-objective evaluation is performed and the results indicate that John is the most valuable to the firm, with Susan a close second.

Subsequently, a new employee is hired, who is very knowledgeable about the use of PC's, but not quite as knowledgeable as John. John is superior with respect to the new employee in all other objectives as well. Since John dominates the new employee, the new alternative is "irrelevant", and according to MAUT practitioners, "should" not affect the ranking of the pre-existing employees. Is this necessarily reasonable? Since John's *relative* value to the firm has been diminished "should" John still be more valuable to the firm than Susan? We would conclude no!

To see that a *prohibition* of rank reversal in this evaluation is *not* reasonable, suppose more and more (similar) "irrelevant" alternatives are hired. Surely there would come a point where the value of John's ability with PC's would be diluted to the point where Susan would be considered to be the most valuable employee. This example, typical of many evaluations, leads us to conclude that *value* is *relative* in many evaluations, and that a methodology that allows for rank reversal is desirable in these situations. Conversely, methodologies, such as MAUT, which preclude rank reversal can produce flawed results for situations where worth *is* affected by relative abundance (above and beyond the impact of affecting the relative importance of the objectives.) This conclusion is not new! The need for a methodology to allow for rank reversal has long been recognized.

For example, Huber and Puto[17] (1983), in an article "Market Boundaries and Product Choice: Illustrating Attraction and Substitution Effects", state that:

> "Choice researchers have commonly used two general approaches to account for the way proximity of a new item affects choice. These approaches differ primarily in the way item similarity, as derived from the dimensional structure of the alternatives is assumed to affect the choice process. The first proposition (proportionality) assumes that the new item takes share from existing items in proportion to their original shares (i.e., no similarity effect)."

Proportionality would *preclude* rank reversal. Huber and Puto continue to say:

> "The second proposition (substitutability) assumes that the new item takes share disproportionately from more similar items – i.e., the closer the added item is to existing items in the set, the more it "hurts" them (a negative similarity effect)."

Huber, Payne and Puto (1982)[18] note that:

> "... the similarity hypothesis asserts that a new alternative takes disproportionate share from those with which it is most similar. Researchers have shown that the similarity effect is operant for individual or aggregate choice probabilities."

Substitutability requires that rank reversals be permitted. Decision-makers must decide, and should not be told, which of these two approaches, proportionality (an open system – ideal synthesis) or substitutability (a closed system – distributive synthesis), is relevant to their evaluation. We believe that the substitution effect, is in general, more appropriate for multi-objective evaluations. Huber and Puto argue that:

> "A substitution effect will be more salient where multi-attribute decision-making occurs. It should, therefore, be most apparent in major purchases

[17] Huber, J. and Puto, C., 1983. Market Boundaries and Product Choice: Illustrating Attraction and Substitution Effects, *Journal of Consumer Research* Vol. 10, June, 31 - 44.

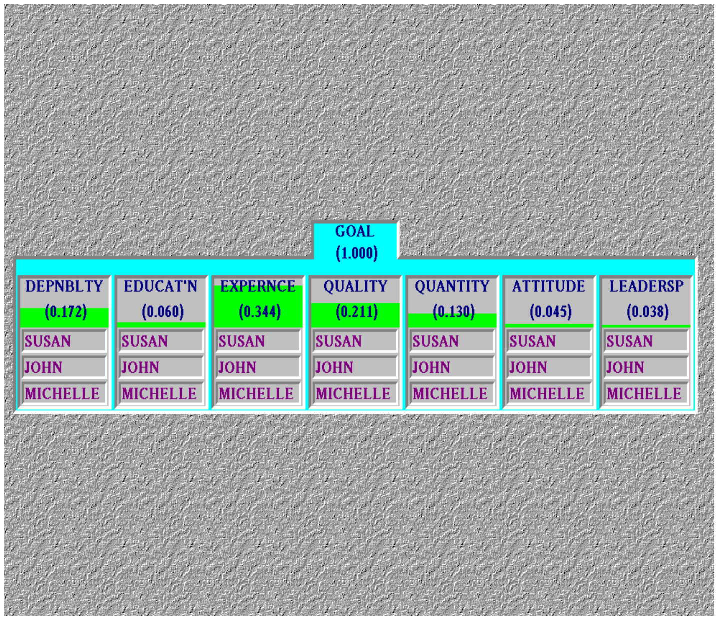
[18] Huber, J., Payne, J. W., and Puto, C., 1982. Adding Asymmetrically Dominated Alternatives: Violations of Regularity and the Similarity Hypothesis. *Journal of Consumer Research*. Vol. 9, June, 90-98.

(where attribute-based processing is more cost effective) and in product classes for which a limited number of attributes emerge that permit easy comparisons across alternatives."

Since Multi Criteria Decision-making (MCDM) and Multi Objective Decision-making (MODM) techniques like AHP are now facilitating comparisons across alternatives for more than just a limited number of attributes, the substitution effect should become even more common and the ability of a methodology to allow rank reversal should be welcomed.

Conversely, there are situations where a rank reversal would *not* coincide with the underlying preferences of the decision-makers. Suppose, for example, that a decision-maker, considering whether to buy an IBM PC compatible or an Apple MacIntosh, has decided on the IBM PC compatible. The introduction of another IBM PC compatible that is not as good on any dimension as the original PC compatible would not, for most decision makers, change the original ranking. For situations such as this, the AHP open system (ideal synthesis) should be used and will not allow a rank reversal. The following example illustrates the differences between the closed system (distributive synthesis) and open system (ideal synthesis).

Employee Evaluation



Abbreviation	Definition
ATTITUDE	ATTITUDE
DEPNBLTY	DEPENDABILITY
EDUCAT'N	EDUCATION
EXPERNCE	EXPERIENCE (INCLUDING PC HARDWARE AND SOFTWARE)
JOHN	Particularly good with computers
LEADERSP	LEADERSHIP
MICHELLE	Leadership her best attribute
QUALITY	QUALITY OF WORK
QUANTITY	QUANTITY OF WORK
SUSAN	Excellent in all categories

(Priorities shown are 'Local' -- relative to parent node.)

Figure 12 – Employee Evaluation with Three Alternatives

Employee Evaluation using the AHP Closed System (Distributive Synthesis)

Figure 12 contains a model used in an AHP evaluation of three employees. With the closed system (distributive synthesis) the “global” priority of each alternative “node” is the product of the node’s local priority with it’s parent’s global priority. The overall priority for each employee is the sum of the employee’s global priorities throughout the model, as shown in Figure 13. In practice there may be several additional levels in an employee evaluation model. Overall, John is the most valuable employee, primarily because of his experience with personal computer hardware and software.

Employee Evaluation

Synthesis of Leaf Nodes with respect to GOAL
Distributive Mode
OVERALL INCONSISTENCY INDEX = 0.1

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5
EXPERNCE=.344				
	JOHN =.163			
	SUSAN =.091			
	MICHELLE=.091			
QUALITY =.211				
	SUSAN =.089			
	JOHN =.067			
	MICHELLE=.055			
DEPNBLTY=.172				
	SUSAN =.074			
	JOHN =.057			
	MICHELLE=.041			
QUANTITY=.130				
	SUSAN =.051			
	JOHN =.040			
	MICHELLE=.040			
EDUCAT'N=.060				
	SUSAN =.025			
	JOHN =.018			
	MICHELLE=.018			
ATTITUDE=.045				
	SUSAN =.018			
	JOHN =.014			
	MICHELLE=.012			
LEADERSP=.038				
	SUSAN =.015			
	MICHELLE=.013			
	JOHN =.011			

Figure 13 – Closed System (Distributive Synthesis) and Overall Priorities

Adding an Irrelevant Alternative

Suppose an additional employee, Bernard, joins the company and suppose Bernard is an irrelevant alternative in that he is dominated on every objective. However, Bernard also has experience with PC hardware and software. Since John's *relative* value to the firm has been diminished, "should" John still be more valuable to the firm than Susan? We would conclude no! This is the result in the model with Bernard added as is shown in Figure 14 and Figure 15.

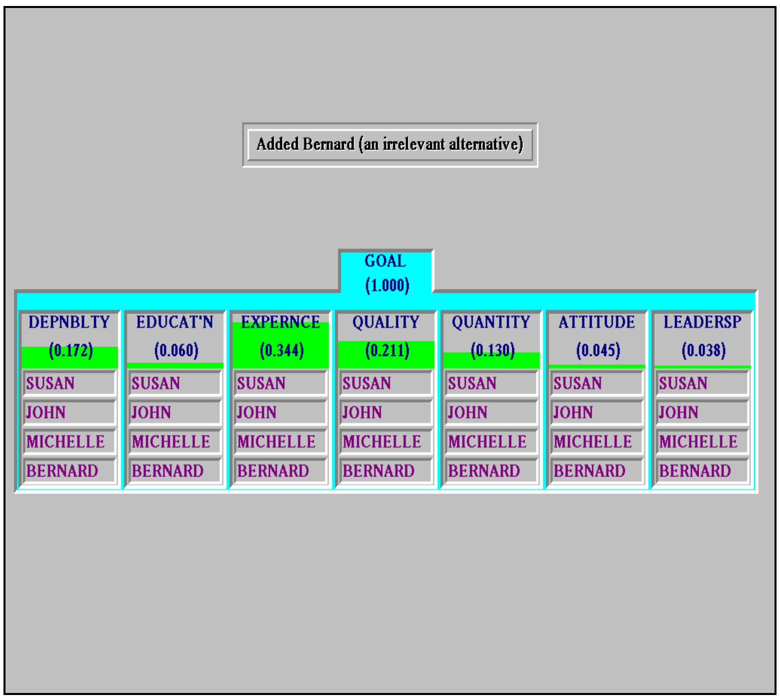


Figure 14 – Added Bernard (an irrelevant alternative)

Added Bernard (an irrelevant alternative)

Synthesis of Leaf Nodes with respect to GOAL
Distributive Mode
OVERALL INCONSISTENCY INDEX = 0.04

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5
EXPERNCE=.344				
	JOHN =.119			
	BERNARD =.093			
	SUSAN =.066			
	MICHELLE=.066			
QUALITY =.211				
	SUSAN =.080			
	JOHN =.060			
	MICHELLE=.050			
	BERNARD =.020			
DEPNBLTY=.172				
	SUSAN =.060			
	JOHN =.046			
	MICHELLE=.033			
	BERNARD =.033			
QUANTITY=.130				
	SUSAN =.047			
	JOHN =.036			
	MICHELLE=.036			
	BERNARD =.010			
EDUCAT'N=.060				
	SUSAN =.022			
	JOHN =.016			
	MICHELLE=.016			
	BERNARD =.006			
ATTITUDE=.045				
	SUSAN =.017			
	JOHN =.013			
	MICHELLE=.011			
	BERNARD =.004			
LEADERSP=.038				
	SUSAN =.013			
	MICHELLE=.011			
	JOHN =.009			
	BERNARD =.004			

Figure 15 – Closed System (Distributive Synthesis) after Adding an Irrelevant Alternative

Employee Evaluation using the AHP OPEN System (Ideal Synthesis)

In an open system (ideal synthesis), the most preferred alternative under each objective receives the full priority of the objective. Each of the other alternatives receives a priority proportional to its preference relative to the most preferred alternative. After the alternatives receive priority from each of the objectives, a subsequent normalization is performed so that sum of all of the alternatives' priorities is equal to 1.0 as can be seen in Figure 16.

The priorities in this example are such that the rank order of the four alternatives with the open system (ideal synthesis) are the same as with the closed system (distributive synthesis) shown in Figure 15. However, if we now remove Bernard, the "irrelevant alternative", and perform an open system (ideal synthesis), Susan remains the most valuable of the three employees (see Figure 17). The result is different from the closed system (distributive synthesis) where John, because of his unique abilities with computers, is the most valuable among the three employees.

Added Bernard (an irrelevant alternative)

Synthesis of Leaf Nodes with respect to GOAL
Ideal Mode
OVERALL INCONSISTENCY INDEX = 0.04

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5
EXPERNCE=.344				
	JOHN =.344			
	BERNARD =.268			
	SUSAN =.191			
	MICHELLE=.191			
QUALITY =.211				
	SUSAN =.211			
	JOHN =.158			
	MICHELLE=.132			
	BERNARD =.053			
DEPNBLTY=.172				
	SUSAN =.172			
	JOHN =.134			
	MICHELLE=.096			
	BERNARD =.096			
QUANTITY=.130				
	SUSAN =.130			
	JOHN =.101			
	MICHELLE=.101			
	BERNARD =.029			
EDUCAT'N=.060				
	SUSAN =.060			
	JOHN =.043			
	MICHELLE=.043			
	BERNARD =.017			
ATTITUDE=.045				
	SUSAN =.045			
	JOHN =.035			
	MICHELLE=.030			
	BERNARD =.010			
LEADERSP=.038				
	SUSAN =.038			
	MICHELLE=.032			
	JOHN =.027			
	BERNARD =.011			

Figure 16 – Open System (Ideal Synthesis) with Four Alternatives

Employee Evaluation

Synthesis of Leaf Nodes with respect to GOAL

Ideal Mode

OVERALL INCONSISTENCY INDEX = 0.1

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5
EXPERNCE=.344				
	JOHN =.344			
	SUSAN =.191			
	MICHELLE=.191			
QUALITY =.211				
	SUSAN =.211			
	JOHN =.158			
	MICHELLE=.132			
DEPNBLTY=.172				
	SUSAN =.172			
	JOHN =.134			
	MICHELLE=.096			
QUANTITY=.130				
	SUSAN =.130			
	JOHN =.101			
	MICHELLE=.101			
EDUCAT'N=.060				
	SUSAN =.060			
	JOHN =.043			
	MICHELLE=.043			
ATTITUDE=.045				
	SUSAN =.045			
	JOHN =.035			
	MICHELLE=.030			
LEADERSP=.038				
	SUSAN =.038			
	MICHELLE=.032			
	JOHN =.027			

Figure 17 – Open System Synthesis After Removing the Irrelevant Alternative

Summarizing, in this example (contrived for illustrative purposes), if there were only three alternatives, John would rank first using the closed system (distributive synthesis) while Susan would rank first using the open system (ideal synthesis). Since, in this illustration, the value of the employees is affected by relative abundance or scarcity of their talents (or in other words, one employee's talents can be *substituted* for another), the closed system (distributive synthesis) is more appropriate and *John should be the most valuable*! Thus, the *lack* of rank reversal with the open system (ideal synthesis) produces "flawed results".

While the rank order of the alternatives with the open system synthesis is the same for three or four alternatives, the rank order of Susan and John is different with the closed system (distributive synthesis) depending on whether Bernard is or is not included. This is due to a dilution effect caused by the Bernard, an "irrelevant alternative". The removal of Bernard causes the priority of each objective to be concentrated (the converse of a dilution of priorities when alternatives are added) under the closed system (ideal synthesis) because the total priority under each objective does not change. Furthermore, the concentration is not the same for all objectives. For, according to the similarity effect discussed previously, a new alternative would take a disproportionate share from those with which it is most similar, so conversely, the removal of an alternative would give a disproportionate share to those with which it is most similar. Since Bernard was most similar to John on the experience objective, John gains proportionately more priority if Bernard is removed and consequently John is the most preferred among the three alternatives. This makes intuitive sense since the value of John's experience with PC hardware and software has increased because it is "more" scarce if Bernard is not included.

When is scarcity germane?

When scarcity is germane a closed system (distributive synthesis) is appropriate and when it is not an open system (ideal synthesis) is appropriate. Yet, it may not always be obvious when scarcity is germane. The following questions can help determine whether to use the distributive synthesis or ideal synthesis:

Q: Is the purpose of the model to forecast, prioritize alternatives, or choose one alternative?

If the model purpose is to forecast or prioritize alternatives, then the closed system (distributive synthesis) is appropriate. If, however, the model purpose is to choose one alternative, then a subsequent question can be posed:

Q: Will alternatives not chosen still be relevant, i.e., will they still matter to you?

If the answer is yes, then the closed system (distributive synthesis) is appropriate. If not, then the open system (ideal synthesis) is appropriate.

How Significant is the Choice of Synthesis Mode in Practice? We investigated forty-four applications of the Analytic Hierarchy Process, applying the above questions about scarcity to each. In our judgment, scarcity was relevant in sixteen of the forty-four applications. Next, we compared the results of a distributive synthesis *and* an ideal synthesis for each application. Of the forty-four applications, thirty six had identical rankings of alternatives regardless of the synthesis mode. Of the remaining eight applications, six had the same first choice. The two applications for which the different synthesis modes produced different 'best' alternatives were each identified as 'closed' systems (for which the original AHP distributive synthesis was appropriate).

Summary

A multi-objective modeling approach must be able to accommodate both "closed" systems – with a fixed amount of resources and where scarcity is germane, and "open" systems – where resources can be added or removed and where scarcity is not germane. The choice of an open or closed system (distributive synthesis or ideal synthesis) for a particular prioritization, choice, or resource allocation problem is one that must be made by the decision makers – not prescribed by a methodology or its axioms. Recognizing that there are situations in which rank reversals are desirable and other situations in which they are not, a logical conclusion is that any decision methodology that always allows or always precludes rank reversals is inadequate. AHP is capable of deriving ratio scale priorities for both types of situations.

Structural adjustment / Product adjustment

Structural adjustment

Structural Adjustment. In some applications you may want to adjust the priorities of a set of nodes based on the structure of the EC tree. When the grandchildren of the current node are the alternatives, a structural adjustment *can* be made to the priorities of the children of the current node based on the total number of grandchildren. The effect of this command is to prevent diluting the importance of a grandchild simply because it has many siblings.

Consider the problem of an elderly widow who is about to write her last will and testament to provide (indirectly) for her grandchildren via her children. Disregarding preferences for children and grandchildren for the moment, the following question might arise. Should she give equal shares of her estate to each of her children, who will then distribute their shares to the grandchildren, or should her children's shares be determined by the number of grandchildren each produced?

Suppose the grandparent (the current node) wishes to leave each grandchild the same amount of money and favors each of her own children equally. If the grandparent were to leave each of her own children equal amounts of money to be passed on to the grandchildren, then the grandchildren would not receive equal amounts (see Figure 18). Those from the larger families would be penalized. An equitable distribution to the grandchildren could be achieved by adjusting the amount each child gets by the number of grandchildren they produced. Children who have produced more grandchildren get a larger priority (or amount of money) (see Figure 19).

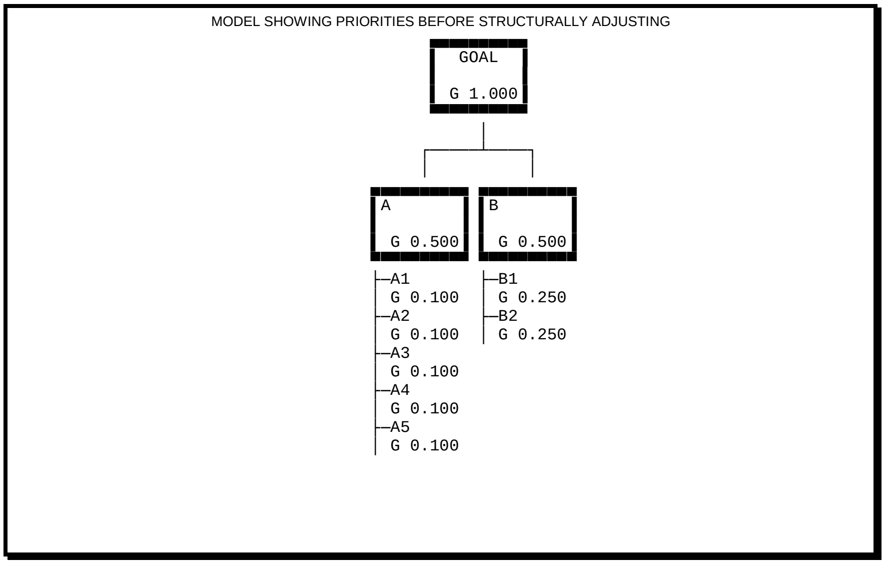


Figure 18 – Before Structurally Adjusting

Whether one uses the structural adjust feature depends on the objectives of the decision maker(s). If the widow's intent is to distribute equal shares to her children, then she would not use the structural adjust. If her intent is to distribute equal shares to her grandchildren, then she would use the structural adjust.

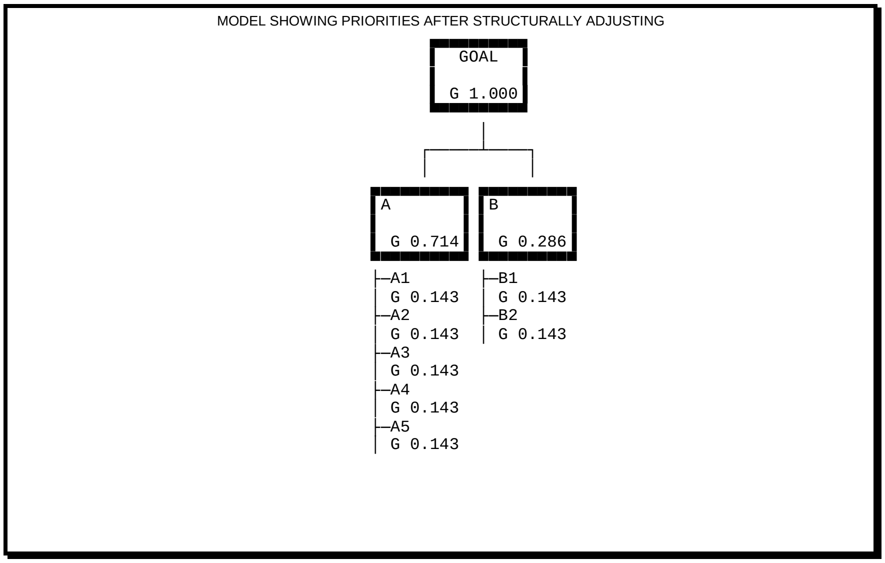


Figure 19 – After Structurally Adjusting

The discussion above assumes no preferences among the children, solely for illustrative purposes. The structural adjustment is actually made after the priorities of the children are derived when you one of the assessment methods.

Product Adjustment

Because all AHP priorities are ratio scale measurement level, it is mathematically meaningful to multiply or divide them. For example, priorities from a benefits *model* can be divided by priorities from a cost *model*, as discussed on page 183. In some applications you may wish to form the product of the priorities from one set of *judgments* with those of another set of *judgments*. For example, you may have scenarios to be judged in terms of both likelihood and importance. You might first make comparisons about the likelihood of the scenarios and subsequently make comparisons about the importance of these same scenarios. While entering the second set of judgments, you would turn on the Expert Choice product switch. After all judgments were entered, the priorities would be calculated as usual. However, if the product adjust switch is on these priorities will be multiplied by the previously calculated priorities (the likelihoods) and normalized appropriately. Thus the priorities of the scenarios would be based on both likelihood and importance.

Complete Hierarchy

An example of a complete hierarchy, as usually depicted in the AHP literature, is shown in Figure 20. A complete hierarchy is complete in the sense that the same objectives appear under each of the scenarios[19]. In this example priorities would be established for the scenarios by making judgments about the relative likelihood of the scenarios. Priorities would be established for the objectives by making pairwise comparisons about the importance of the objectives, given each of the scenarios. And priorities would be established for the alternatives by making judgments about the relative preference of the alternatives with respect to each of the objectives.

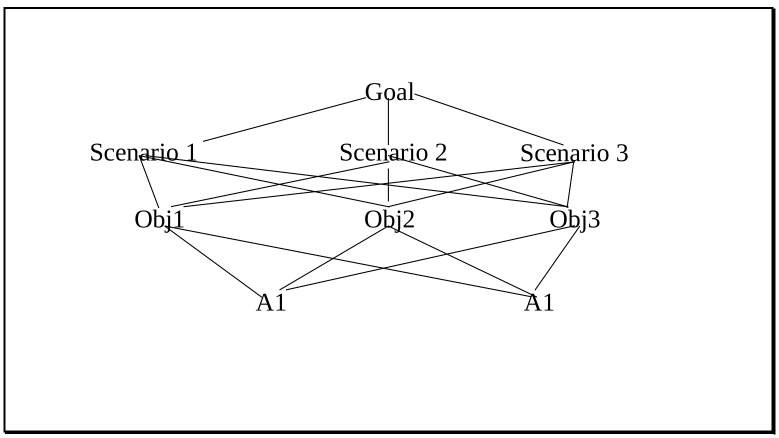


Figure 20 – Complete Hierarchy

[19] Or if objectives appear below the goal, and the same sub-objectives appear under each of the objectives, the hierarchy would be said to be complete.

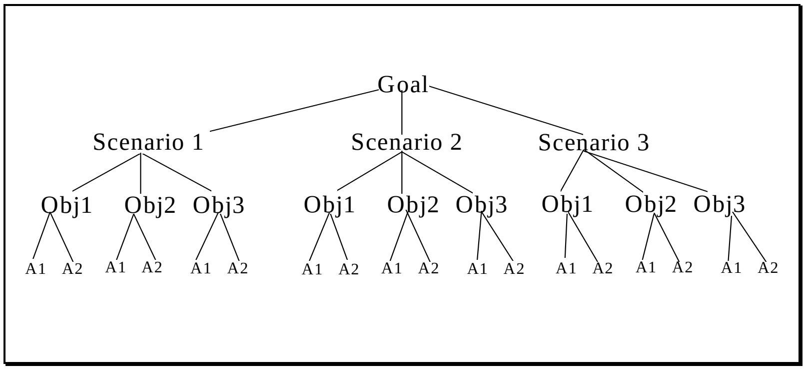


Figure 21 – Incomplete Hierarchy

An incomplete hierarchic representation of this model is presented in Figure 21. Although the hierarchy is still complete, it need not be – different objectives (or some the same and some different) could appear under each of the scenarios. The incomplete representation of this hierarchy entails another difference – that is dependence. Whereas the pairwise comparisons of the alternatives with respect to each of the objectives in Figure 20 are independent of the scenario, they are dependent in Figure 21. This dependency has two ramifications. First, it allows for more specific judgments to be made. For example, when comparing the relative preference of investment alternatives with respect to appreciation, the relative preference of a real estate investment compared to stocks could be much different for a scenario of low interest rates than for a scenario of high interest rates. The second ramification is that many more judgments must be made using the incomplete hierarchic approach.

Expert Choice represents hierarchies in the more general, incomplete form as shown in Figure 21. If however, one would like to make judgments about the alternatives irrespective of the scenarios, judgments need be made in only the leftmost plex (the nodes below SCENARIO1) and the Expert Choice complete hierarchy command[20] will replicate the plexes below the

---

[20] Invoked from the OBJ1 node below SCENARIO1.

objectives under SCENARIO1 to appear below the objectives under each of the other two scenarios.[21]

Benefit/Cost

An alternate approach to an 'achievement of objectives' framework for making decisions is to consider benefits and costs separately. Organizations have been doing 'benefit/cost' studies for quite some time. Such studies have often faltered because of the difficulty in quantifying benefits. (Costs were much easier to quantify). Our ability to use AHP to quantify *all* benefits, including those that are qualitative, and *all* costs, including qualitative costs, brings new life to benefit cost analysis.

The typical AHP model includes both benefits and costs. The *objectives* in the typical AHP model, can, if desired, be viewed in terms of *benefits* and *costs*. For example, in choosing a car, we might consider both performance and maintenance requirements as objectives. The former is clearly a benefit, while the latter is a cost. When we pairwise compare alternative cars with respect to these objectives, we ask which car is more *preferable*. A high performing car is more preferable than a low performing car. A car requiring low maintenance is more preferable than one requiring high maintenance. By asking which is more preferable, we implicitly view objectives as benefits – high performance is a benefit and low maintenance is a benefit.

Instead of including benefits and costs in a single hierarchy, a benefit/cost approach can be taken in which one hierarchy is used to measure the benefits of the alternatives, and a second hierarchy, used to measure the costs of the alternatives. The two hierarchies are similar only in that they have the same alternatives. The benefit priorities from the benefits hierarchy are subsequently divided by the cost priorities from the cost hierarchy to give a measure of benefit/cost ratio.[22]

---

[21] More generally, additional levels can appear between the objectives and the alternatives. If judgments are made for the entire plex below SCENARIO1, as well as for the objectives below SCENARIO2 and SCENARIO3, the complete hierarchy command can then be invoked from the first objective below SCENARIO1.

[22] This is mathematically meaningful because each hierarchy produces ratio scale priorities and the ratio of ratio scale numbers is a ratio scale.

Alternatives in the benefits hierarchy are evaluated such that those giving the highest benefit receive the highest priority, as is customary. In the cost hierarchy, alternatives with the higher costs should receive the highest priority, because we are trying to measure costs. To do this, we either implicitly or explicitly ask which of two alternatives is more ‘costly’ rather than more ‘preferable’ as is customary when benefits and costs are treated in one model.

The following example is a model to choose whether to build a bridge, or a tunnel across a river to continue using an existing ferry[23]. The factors involved in the benefits and costs of crossing a river are given in the two hierarchies below. They fall into three categories: economics, social and environmental. The decision is made in terms of the ratio of benefits priority to costs priority for each alternative.

The benefits hierarchy:

Determine way to cross the river yielding greatest benefits

```
                     ╔TIME════
                     ╠INCOME═══
           ╔ECONOMIC═╬COMMERCE═
           ║         ╠LOCALCOM═
           ║         ╚JOBS═════     \╔BRIDGE (0.57)
GOAL═══════╣         ╔SAFE/REL═      ╠TUNNEL (0.36)
           ╠SOCIAL═══╬COMMUNIC═     /╚FERRY  (0.07)
           ║         ╚PRIDE════
           ║         ╔COMFORT══
           ╚ENVIRMNT═╬ACCESSIB═
                     ╚AESTHETC═
```

The alternative with the greatest benefit is the bridge, and the alternative with the least benefit is the ferry.

---

[23] Saaty, Thomas L. and Forman, Ernest H., *The Hierarchon - A Dictionary of Hierarchies*, The Analytic Hierarchy Process Series - Volume V, RWS Publications, Pittsburgh PA, 1993, p. A21.

The costs hierarchy:

Determine the most costly alternative for crossing the river

```
                  ╔CAPITAL═════
         ╔ECONOMIC═╬MAINTN'C═
         ║         ╚DISRUPT'N═════
GOAL═════╣          ╔LIFESTYL═      \╔BRIDGE (0.36)
         ╠SOCIAL═══╬PEOPLE═         ╠TUNNEL (0.58)
         ║         ╚DISLOCAT═══    /╚FERRY  (0.05)
         ║          ╔AUTOEMIT══
         ╚ENVIRMNT═╬POLLUT'N═
                    ╚ECOLOGY═
```

The alternative with the highest cost is the tunnel, and that with the least cost is the ferry, since it is already built.

For this example we have the following ratios of benefits/costs priorities:

Bridge 0.57/0.36 = 1.58
Tunnel 0.36/0.58 = 0.62
Ferry 0.07/0.05 = 1.28

This evaluation shows the best choice is to construct a bridge across the river as Bridge has the largest benefit to cost ratio. Note that this has taken into consideration the capital requirements in terms of dollars as well as other benefits and costs not measurable in dollars. The ferry, an existing alternative, serves as a reference point. Since the bridge benefit/cost ratio is higher than that of the ferry it gives additional support to the choice of the bridge.

Why use benefit cost ratio?

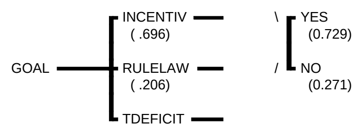
Decisions are often based on benefit/cost ratios without much thought as to what guarantees that the alternative with the highest benefit cost ratio is also the most preferred alternative. Although this is usually true, there are exceptions. Consider the following decision. You can invest in either (not both) of two alternatives, A and B. Alternative A, which requires an investment of $1 will, for certain, return a benefit of $10. Alternative B requires an investment of $50 but will, for certain, return a benefit of $100. The benefit cost ratios of alternatives A and B are 10 to 1 and 2 to 1 respectively. Clearly, Alternative A has the higher benefit cost ratio while Alternative B is the better investment! In this example, as well as in general, it makes more sense to consider the net benefit from each alternative and choose the alternative with the highest net benefit, rather than the alternative with the highest benefit cost ratio

Sorting and selecting alternatives based on their benefit/cost ratios is a useful technique when considering the selection of a combination of alternatives, subject to a budget constraint. But here again, this approach can produce an inferior choice than a maximization of benefit approach. The latter is, however, more difficult to calculate, although it can be done with not too much difficulty using today's technologies including personal computers and optimization algorithms such as linear programming and integer programming. The optimization approach is also more flexible than the benefit/cost ratio approach, since it can consider a variety of constraints in addition to a total budget constraint. In practice, the benefit/cost ratio approach usually, although not always, produces results similar to the optimization approach. A detailed discussion of the optimization approach and a comparison to the benefit/cost approach begins on page 235.

Benefit/Cost/Risk

Risk is another multiplicative factor that can be incorporated when evaluating alternatives. When evaluating R&D projects for example, consideration of the benefits without consideration of the risks is meaningless. A benefit can be certain, or uncertain. When benefits are uncertain we can estimate probability distributions (as discussed in the

Benefits of Trade Sanctions with China



Abbreviation	Definition
GOAL	
INCENTIV	Maintain Incentive to make and sell in China
NO	Don't invoke trade sanctions
RULELAW	Rule of Law - Bring China to responsible free trading
TDEFICIT	Help trade deficit with China
YES	Invoke trade sanctions

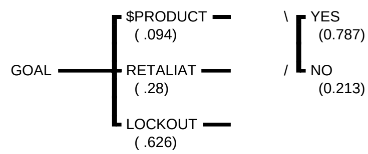
Figure 22 – Benefits of Trade Sanctions

forecasting section beginning on page 213) calculate expected benefits, and use expected benefits[24] in the evaluation of alternatives. Another approach is to use a separate hierarchy to estimate the risks of the alternatives, and then calculate benefit/(cost x risk). The following example[25] of the consideration of a trade war with China over intellectual property rights illustrates the benefits/costs/risk approach. In February of 1995, the U.S. was considering whether or not to sanction China because of Chinese piracy of U.S. technology and management know-how. Three simple hierarchies were used to assess the relative benefits, costs and risks of implementing trade sanctions as seen in Figure 22, Figure 23, and Figure 23 below:

[24] Variances can also be calculated and used as a consideration when evaluating alternatives.

[25] Saaty, Thomas L., *Decision Making with Dependence and Feedback*, RWS Publications, Pittsburgh, PA, 1996, p 44.

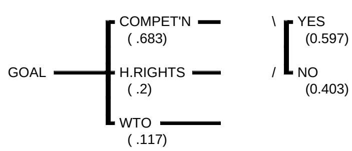
Costs of Trade Sanctions with China



Abbreviation	Definition
GOAL	
$PRODUCT	$Billion Tarrifs make Chinese products more expensive
LOCKOUT	Being locked out of big infrastructure buying: e.g., airports
NO	Don't invoke trade sanctions
RETALIAT	Retaliation
YES	Invoke trade sanctions

Figure 23 – Risks of Trade Sanctions

Risks of Trade Sanctions with China



Abbreviation	Definition
GOAL	
COMPET'N	Long term negative competition
H.RIGHTS	Effect on human rights and other issues
NO	Don't invoke trade sanctions
WTO	How to justify China joining WTO
YES	Invoke trade sanctions

Figure 24 – Risks of Trade Sanctions

Figure 25 – Benefit/(Cost * Risk) Ratios

The benefits of imposing trade sanctions are greater than not imposing trade sanctions. However, the costs and risks of imposing trade sanctions are also greater than not imposing trade sanctions. Because benefits, costs and risks are all ratio scale measures, we can calculate the benefit/(cost*risk) ratio as seen in Figure 25. Clearly, it is better not to impose trade sanctions. This result of this evaluation by Thomas Saaty and Jen Shang was sent to several congressmen as well as the chief U.S. negotiator, Mickey Kantor. Mr. Kantor's office subsequently called Dr. Saaty congratulating him on the outcome, which coincided with the agreement reached between the United States and China.

Pairwise Comparisons versus MAUT Swing Weights

The ability for people to understand and make pairwise comparisons is one of AHP's major strengths. To appreciate this, we invite the reader to compare AHP's judgment process described on pages 72 and 77 with that used by advocates of another theory, Multi Attribute Utility Theory

(MAUT) advocates, such as Kirkwood[26], to derive weights for the objectives directly above the alternatives in a decision hierarchy.

> …First, the single dimensional value functions have been specified so that each of them is equal to zero for the least preferred level that is being considered for the corresponding evaluation measure. Similarly, each of the single dimensional value functions has been specified so that it is equal to one for the most preferred level that is being considered for the corresponding evaluation measure.
>
> From these properties of the single dimensional value functions, it follows that the weight for an evaluation measure is equal to the increment in value that is received from moving the score on that evaluation measure from its least preferred level to its most preferred level. This property provides a basis for a procedure to determine the weights. … Specifically these steps are as follows:
>
> Consider the increments in value that would occur by increasing (or 'swinging") each of the evaluation measures from the least preferred end of its range to the most preferred end, and place these increments in order of successively increasing value increments.
>
> Quantitatively scale each of these values as a multiple of the smallest value increment.
>
> Set the smallest value increment so that the total of all the increments is 1.
>
> Use the results of Step3 to determine the weights for all the evaluation measures.

A paraphrased description of Kirkwood's application of these rules to his prototype example is:

> Suppose that the swing over the total range for Productivity Enhancement (Objective 1) has the smallest increment of value.… Suppose further that the swing over the total range for Cost Increase (Objective two) from 150 to 0 has 1.5 times as great a value increment as the swing over Productivity Enhancement from -1 to 2, and the swing over Security (Objective 3) from -2 to 1 has 1.25 times the value increment of swing over Cost increase.

---

[26] Kirkwood, C. 1997. "Strategic Decision Making – Multiobjective Decision Analysis with Spreadsheets", Duxbury Press, Belmont CA,

An alternate approach to determining weights used by MAUT is even more obscure. It involves the consideration of hypothetical alternatives and asks the decision maker to determine intermediate levels for the alternative for which they would be indifferent between that alternative and one at its maximum level. Thus instead of the swing weight questions described above, the dialog might be something like:

> "Consider a hypothetical alternative that has the least preferred level for all the evaluation measures. Now suppose that you could move one and only one from its least preferred level to its most preferred level. Which would you move? Now suppose you could not move that one, which is the next one you would move? Now suppose that you could either move the second from its most least preferred level to its most preferred level, or the first from its least preferred level to some intermediate level. Select the intermediate level for which you would be indifferent between the two possibilities." (This question is usually easiest to answer by considering a specific intermediate level for Security, and then adjusting this level until indifference is established).

Swing weights and other techniques such as balance beams are, in our opinion much more difficult to understand and apply then the pairwise comparisons used in AHP.

Integrating with Quantitative Methodologies

Although quantitative models, such as those characterized as Operations Research / Management Science models, have provided substantial benefits to corporations and governments over the past half century and more, many roadblocks have prevented the realization of their ultimate potential in the decision-making process. One roadblock, the easy access to required computational resources has been eliminated – it is now possible to solve very large quantitative problems on desktop personal computers. Another roadblock remains. Decision-makers often lack the understanding and ability to integrate quantitative models' results with other concerns – both qualitative and quantitative. Most quantitative models provide a 'solution' to a problem formulated to represent the 'real world'. The solution is usually framed from only one of many perspectives. Without a synthesizing mechanism like AHP, decision-makers are left to either 'go with the model results' or with their intuition. Most often, they go with their intuition. By helping decision-makers synthesize the results from one or even several quantitative models with their intuition, AHP enhances the value of quantitative models. In this section we will illustrate the integration of AHP with a variety of quantitative models including linear programming, queueing, critical path method, forecasting, and integer programming.

Linear Programming

The idea for a new product must be developed into specifics. There are usually numerous alternatives for designing each "piece" of a product, and the problem of choosing the "best" design from a very large combination of alternatives can be overwhelming. Traditional textbook examples illustrate how linear programming can be helpful in selecting the best combination of components for a product. Consider a problem of selecting plastic body materials for a new Sporty Convertible being designed by an auto manufacturer. A traditional linear programming formulation might consist of an objective function to minimize costs, subject to constraints on:

- body weight – that the body weight be no more than 120 pounds,

- coverage – that there be at least 5 cubic feet of body material in order to cover the body, and
- strength – that the mixture of materials possess a strength of at least 100 pounds per square foot.

The linear programming solution to this problem would be to use 4.5 cubic feet of the standard material, .174 cubic feet of the super material, and .326 cubic feet of the lightweight material, for a total cost of $545, with a weight of 120 pounds and a strength of 100 pounds per square foot.

Table 2 – Material Characteristics

Material Description	1 Standard	2 Super	3 Econo	4 Strong	5 Lightwt
Cost/cu. Ft.	105	220	85	103	107
Weight/cu . ft.	25	15	40	55	15
Strength lbs/Sq. ft.	20	35	11	42	12

Let's reflect on where some of the relationships and parameters for this model would come from in a real world application. Minimizing cost is obviously an objective, and the coefficients of the objective function, representing the cost per cubic foot of material can be obtained from a data base of suppliers. The fact that we "must" have at least 5 cubic feet of body material to cover the body can come from preliminary design drawings of the sport convertible.

But the constraint requiring that the body weigh no more than 120 pounds is somewhat contrived since one might argue that a light body weight is an "objective" (rather than a constraint) and that we really do not know what we "must" have as a maximum body weight. We would like the body to weigh as little as possible so that the car will accelerate better and use less fuel. Then why was body weight represented as a constraint in the traditional formulation? Simply as a convenience, because linear programming allows only one objective, and we had already chosen cost minimization as that objective.

Similarly, the desire to have as strong a body material as possible is an objective, not a constraint. Almost every real world decision involves multiple objectives. Many of the constraints in LP problem formulations are actually objectives in disguise[27] and are included because LP formulations are limited to one objective. In the Sporty Convertible example, we had already chosen cost minimization as that objective. Thus we attempted to achieve a weight objective by including a constraint that the body material must weigh no more than 120 pounds. However, this approach is not really adequate. Specifying a value of 120 is somewhat arbitrary. Why not 100, or 150?

Because many of the constraints in LP problem formulations are actually objectives in disguise, a pure linear programming approach to this problem appears to be "forced" and there is strong likelihood that senior management will, rightfully, feel uncomfortable with the analysis and not make proper use of it in their decision.[28] [29]

Let's see how a decision-maker might actually approach this decision. It would be highly unlikely that he would begin with a linear programming formulation in mind. Instead, he might query a database of body materials and be presented with the following information:
He might then begin to list the pros and cons of each material. For example, the "pro" for the "Econo" material is its low cost per cubic foot. However, it has two "cons": it is relatively heavy, and it is not very strong. The "Lightwt" material's pro is its relatively low weight, but its cons are its moderately high cost (at least higher than the "Econo" material) and its relatively low strength. The "Super" material has two pros: its relatively low weight and relatively high strength. It also has one important con: its very high cost.

---

[27]Some are "both", a constraint on some minimal (or maximal) value, and an objective to achieve as much (or as little) beyond that value.
[28]We believe that this is the major reason that LP has not been used more extensively in practice.
[29]Another approach to overcoming the limitation of only one objective in an LP solution is called Goal programming. See Dyer, Robert F., Forman, Ernest H., and Mustafa, Mohammad, "Decision Support for Media Selection Using The Analytic Hierarchy Process", Journal of Advertising, Volume XXI, Number 1, March 1992 pp 59-72.

The decision-maker might then ponder how to evaluate the tradeoffs between the pros and cons. Using a process such as that described on page 136. The decision maker might identify three primary objectives: (low) cost, (low) weight, and (high) strength. An AHP model with these objectives and

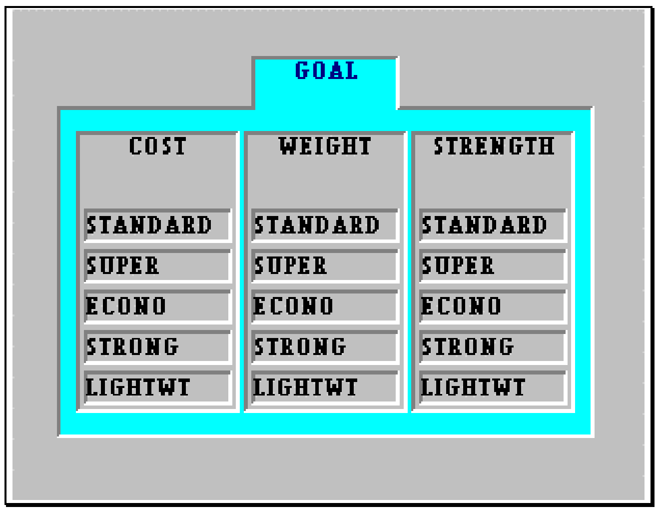


Figure 26 – Design "Best" Sporty Convertible Body Material

the five alternatives is shown in Figure 26.

The relative preferences of the alternatives can be determined by the decision-maker using not only the hard data about the materials, but quite probably with subjective judgments about the utility of the characteristics represented by the hard data as well. For example, when making judgments with respect to the strength objective, the decision maker might refer to Table 2 and, using his previous experience, judge that the "Econo" and "Lightwt" materials are about EQUALLY preferable in spite of the fact that the "Lightwt" is just a little bit stronger; he might then judge that the "Standard" material is STRONGLY more preferable to either, and the "Strong" material is only MODERATELY more preferable to the "Standard" material.

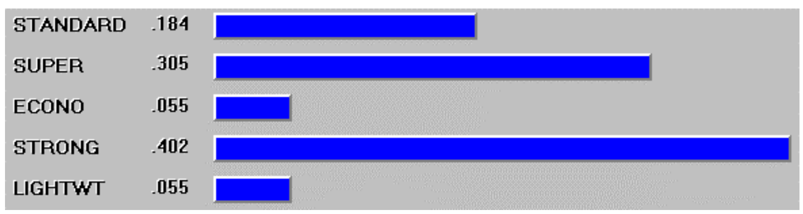


Figure 27 – Priorities with respect to Strength

These judgments and the judgments for other pairs of alternatives with respect to Strength resulted in the priorities shown in Figure 27.

Preferences with respect to Cost and Weight were developed using the Expert Choice Data mode and specifying numerical values equal to the

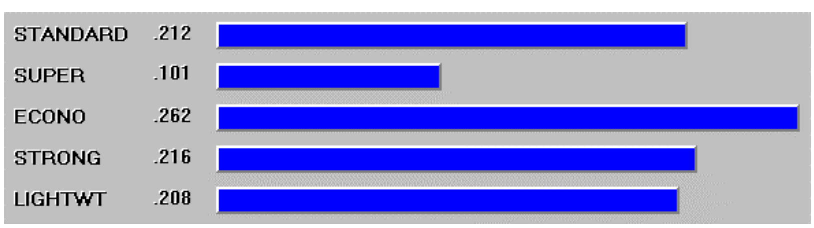


Figure 28 – Priorities with respect to Cost

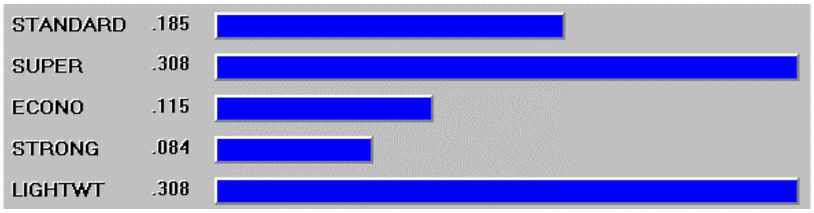


Figure 29 – Priorities with respect to Weight

reciprocals of the costs and weights of the materials.[30] The resulting priorities are shown in Figure 28 and Figure 29.

Next, pairwise comparisons for the relative importance of the three objectives were made with the resulting priorities shown in Figure 30.

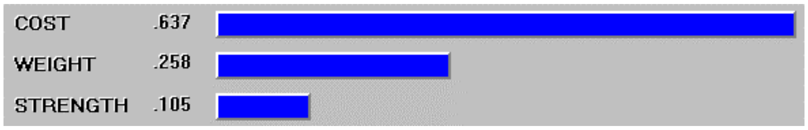


Figure 30 – Priorities of Objectives

Finally, a synthesis of the priorities of the five materials over the three objectives resulted in the priorities shown in Figure 31.

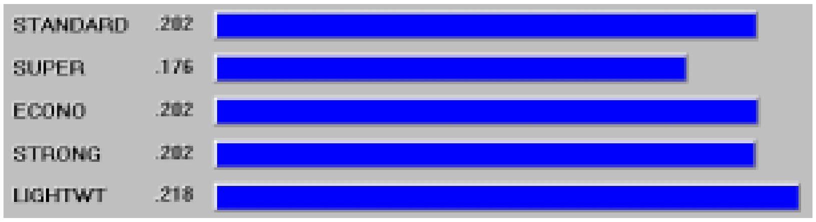


Figure 31 – Overall Material Priorities

[30] Reciprocals are used since low cost and low weight are preferred.

Table 3 – First LP Solution

	Standard	Super	Econo	Strong	Lightweight
Effectiveness	0.202	0.176	0.203	0.202	0.218
Decision Variables	0	0	0	0	5
Effectiveness			1.09		
Cost			535.00		
Weight			75.00		
Strength			60.00		

These priorities represent the overall relative "effectiveness" of the alternatives with respect to the three objectives. It is interesting to observe how close these measures of "effectiveness" are, especially for the top four alternatives, particularly since there were such significant differences with respect to the individual objectives.

Based on these measures of "effectiveness", we can formulate an LP model to determine the composition, at first using only the constraint that we must have five cubic feet of body material to cover the frame. Since the number of basic variables in an LP is equal to the number of constraints, it is not surprising that the "optimal" solution, shown in Table 3, is to use five cubic feet of the plastic with the highest measure of effectiveness, the lightweight plastic.

Now we must examine the solution. Is there anything that appears missing or wrong? If so, we must include additional objectives or constraints. For example, in the above, we have treated low weight and high strength more naturally as objectives, rather than as constraints as in the traditional LP formulation. But if we implement this solution, what would the cost, weight and strength characteristics of the body material be? Examination of Table 3 shows that the cost would be $535, the body weight would be 75 pounds, and the strength would be 60 pounds/per square foot. The latter is well below the 100 pounds originally thought to be the minimum required. So let's now add a constraint specifying that the

Table 4 – Modified LP Solution

	Standard	Super	Econo	Strong	Lightweight
Effectiveness	0.202	0.176	0.203	0.202	0.218
Decision Variables	0	0	0	1.33	3.67

Effectiveness	1.07
Cost	529.67
Weight	128.33
Strength	100.00

minimum strength should be 100 pounds. The modified LP, shown in Table 4, results in an optimum solution of 1.33 cubic feet of the strong and 3.67 cubic feet of the lightweight.

Not only has the strength increased to the required 100 pounds, but the cost has actually decreased from $535 to $529.57. This has been achieved by increasing the body weight from 75 pounds to 128.33 pounds. Since 128.33 body weight is acceptable, this solution is accepted as "optimal".

In comparing this multi-objective solution to that obtained with the traditional single objective LP approach – of minimizing cost subject to (somewhat arbitrary) constraints, it is seen that the multi-objective approach allows us to trade off cost versus weight, since this solution is lower in cost ($529.57 vs. $545.65) but heavier (128.33 lbs. vs. 120 lbs.) In addition, this solution uses a mixture of only two plastics as opposed to three for the traditional solution, a simplification that might result in additional savings as well.

In summary, the multi-objective approach consists of using AHP to derive measures of effectiveness for the alternatives considering more than just the single objective, cost. It then uses LP with only the obvious constraint(s) (in this case body coverage). The tentative solution is then examined to see if it is reasonable. If not, because one or more "must" objectives are obviously not met (in this example an insufficient body strength), new constraint(s) are introduced for the emerging "must(s)" and the LP solved again. In addition, judgments in the AHP model that are used to derive the measures of effectiveness of the alternatives can be re-evaluated in light of the knowledge gained by looking at the tentative

Table 5 – Measures of Performance

Number of Servers	Probability a request will have to wait	Average # of requests in queue	Average time to complete service
1	.95	18.05	20 hours
2	.31	0.28	1.29 hours
3	.01	0.04	1.03 hours

solution. Iteration continues until an "optimal" solution satisfying the multiple objectives is achieved.

Queueing Models

Consider the problem of deciding how many draftsmen with CAD/CAM equipment are needed to design new products. A queueing model provides the following measures of performance based on the number of servers (draftsmen with CAD/CAM equipment):

Having determined this, the question remains, how many servers should be used? The queueing model only helped to derive some measures of effectiveness. It did not really answer the question. A textbook exercise might state that it costs X$ per hour that the request is in queue or in service and then ask which of these three alternatives is best – a simple calculation. Simple, but unrealistic. What does it really 'cost'? One way to begin to answer this question and arrive at a decision is to list the pros and cons of each of the three alternatives as shown in Figure 6.

AHP can help management make the decision on how many draftsmen with CAD/CAM systems to use. The objectives for the decision can be extracted directly from the pros and cons:

Financial considerations
- Salaries of draftsmen
- Expense of CAD/CAM equipment

Morale (and not stifling creativity)
- Engineers
- Draftsmen

Risk of bottlenecks and degradation of service due to
- Absence of draftsmen
- Failure of CAD/CAM equipment
- Increased workload

Exposure to obsolescence of CAD/CAM equipment
- Within one year
- More than one year

Personnel management
- Hiring draftsmen
- Managing draftsmen

A rational decision about how many servers (draftsmen with CAD/CAM equipment) to use must be based on objectives such as these.

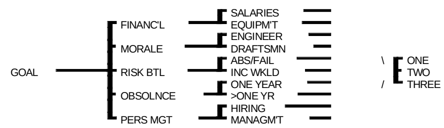
Table 6 – Pros and Cons of Alternative Number of Servers

Altern-ative	PROS	CONS
One Server	Low Expense for draftsmen Low expense for CAD/CAM equipment	Draftsman will see many requests waiting in line and become dejected, feeling that he will never get caught up Engineers will become frustrated because they almost always have to wait for a draftsman to start their job Engineers will become frustrated because the average time to get their job back from the draftsman will be 20 hours, or 2 1/2 days! Risk of machine failure or absent draftsman – what happens if one CAD/CAM machine goes down and or a draftsman does not come in to work?
Two Servers	Draftsmen will see a great reduction in requests waiting in line Engineers will see a great reduction in the wait for the completion of their job –from 20 hours to 1.3 Engineers will have wait for a draftsman to start on their job only about 31% of the time Decreased risk due to machine failure or absent draftsman– if one CAD/CAM machine goes down or a draftsman does not come to work there will still be a backup	Increased expense for draftsmen Increased expense for CAD/CAM equipment Increased exposure to obsolescence. If better machines become available, it is better to have as few on hand as possible so they can be scraped and replaced with new ones Exposure to risk of sharp degradation of service if the workload (arrival rate) increases
Three Servers	Draftsmen will see a further reduction in requests waiting in line Engineers will have to wait for a draftsman to start on their job only	Increased expense for draftsmen Increased expense for CAD/CAM equipment Increased exposure to obsolescence. If better machines become available, it is

	about 1% of the time. Decreased risk due to machine failure or absent draftsman– if one or two CAD/CAM machines goes down or draftsmen do not come in to work there will still be backup(s) Engineers will see some reduction in the wait for the completion of their job –from 1.3 to 1 hour No sharp degradation of service even if the workload (arrival rate) increases	better to have as few on hand as possible so can scrap them to keep up with technology Hiring difficulties for skilled draftsmen that know how to use the CAD/CAM system Managing three draftsmen will require more coordination and effort

The decision will follow from an AHP model (see Figure 32) and judgments about the relative preferences of the alternatives with respect to these objectives and about the relative importance of the objectives. These judgments will be based partly on the results of the queueing model (as elaborated in the pros and cons) and partly on the intuition, knowledge and experience of the decision-maker.

DECIDE THE BEST NUMBER OF DRAFTSMEN -CAD/CAM SYSTEMS



Abbreviation	Definition
GOAL	
>ONE YR	OBSOLESCENCE after 1 year
ABS/FAIL	ABSENCE OF DRAFTSMEN OR FAILURE OF EQUIPMENT
DRAFTSMN	DRAFTSMEN MORALE AND EFFICIENCY
ENGINEER	ENGINEER MORALE, CREATITIVITY AND EFFICIENCY
EQUIPM'T	COST OF CAD/CAM EQUIPMENT
FINANC'L	
HIRING	DIFFICULTY IN HIRING DRAFTSMEN WITH CAD/CAM EXPERTISE
INC WKLD	RISK OF BOTTLENECKS DUE TO INCREASE IN WORKLOAD
MANAGM'T	DIFFICULTY IN MANAGING TWO OR THREE DRAFTSMEN
MORALE	MORALE AND EFFECTIVENSS OF ENGINEERS AND DRAFTSMEN
OBSOLNCE	EXPOSURE TO HAVING INVESTMENT IN OBSOLETE EQUIPMENT
ONE	ONE DRAFTSMAN - CAD/CAM SYSTEM
ONE YEAR	OBSOLESCENCE WITHIN ONE YEAR
PERS MGT	PERSONNEL MANAGEMENT (OF DRAFTSMEN)
RISK BTL	RISK OF BOTTLENECKS DEVELOPING DUE TO ABSENCE/FAILURE
SALARIES	SALARIES OF DRAFTSMEN
THREE	THREE DRAFTSMEN - CAD/CAM SYSTEMS
TWO	TWO DRAFTSMEN - CAD/CAM SYSTEMS

Figure 32 – Deciding on the number of servers

Critical Path Method

Consider the decision that management faces in deciding what resources to apply to a project in order to complete the project in as short a time as is "practically possible". Part of what management will consider practically possible will involve tradeoffs between time, money, labor, and materials.

The Critical Path Method (CPM) is a well-known Operations Research technique used in project management. CPM is useful in analyzing the precedence relationships in a project of many activities, in determining which activities are on the "critical path", and determining how long it will take to complete the project. In addition, CPM can indicate where to apply additional resources if management desires to "crash" activities (speed them up) in order to reduce the total project completion time. CPM provides a great deal of useful information such as the time, cost tradeoffs in Table 7.This information makes it clear that a decision must be made -- management must choose the level of crashing from the available alternatives.

Table 7 – Project Completion Alternatives

Alternative	Months to Complete	Project Cost ($000's)
Alternative 1	35	16,814.00
Alternative 2	34	16,820.67
Alternative 3	33	16,827.33
Alternative 4	32	16,845.33
Alternative 5	31	16,873.00
Alternative 6	30	16,942.00
Alternative 7	29	17,027.00

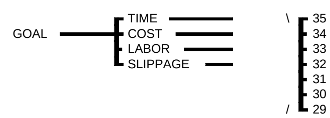
Table 8 – Alternatives and Objectives

Alternative	Months to Complete	Project Cost ($000's)	Project Stress	Likelihood of Slippage
Alternative 1	35	16,814.00	Low	Very Little
Alternative 2	34	16,820.67	Low	Low
Alternative 3	33	16,827.33	Moderate	Low
Alternative 4	32	16,845.33	Moderate	Moderate
Alternative 5	31	16,873.00	Moderate	High
Alternative 6	30	16,942,00	High	High
Alternative 7	29	17,027.00	Very High	Very High

At first, the choice in this example appears relatively easy. The difference in cost between a project completion period of 35 months and 29 months is slightly more than two hundred thousand dollars out of a total of about 17 million dollars. Expressed as a percentage, this is not a large amount. But relative to management's discretionary budget it may be very large. Thus, considering only the cost objective, it is not clear what management would decide. In addition, there are other factors that must be considered when trying to speed up a project, such as increases in labor stress and the probability of project slippage. Thus, management should consider objectives such as those in Table 8 in deciding which of the alternatives to choose:

This decision, as almost all decisions, depends on both quantitative considerations (months to complete and project cost) and qualitative considerations (labor stress, likelihood of slippage, and the relative importance of the four objectives). While the CPM analysis has helped in determining the numerical tradeoffs between time to completion and project cost, it is only a part of the decision support system. An AHP analysis using

DETERMINE BEST LEVEL OF CRASHING



Abbreviation	Definition
GOAL	
29	29 MONTHS AT 17,027,000, V. HIGH STRESS, V. HIGH CHANCE SLIPPAGE
30	30 MONTHS AT 16,942,000, HIGH STRESS, HIGH CHANCE OF SLIPPAGE
31	31 MONTHS AT 16,873,000, MODERATE STRESS, HIGH CHANCE OF SLIPPAGE
32	32 MONTHS, $16,845,330, MODERATE STRESS, MODERATE CHANCE SLIPPAGE
33	33 MONTHS AT $16,827,330, MODERATE STRESS, LOW CHANCE OF SLIPPAGE
34	34 MONTHS AT $16,820,670, LOW STRESS, LOW CHANCE OF SLIPPAGE
35	35 MONTHS AT $16,814,000, LOW STRSS, V. LITTLE CHANCE OF SLIPPAGE
COST	
LABOR	STRESS ON PERSONNEL
SLIPPAGE	LIKELIHOOD OF SLIPPAGE
TIME	TIME TO COMPLETE PROJECT

Figure 33 – AHP Model for Best Level of Crashing

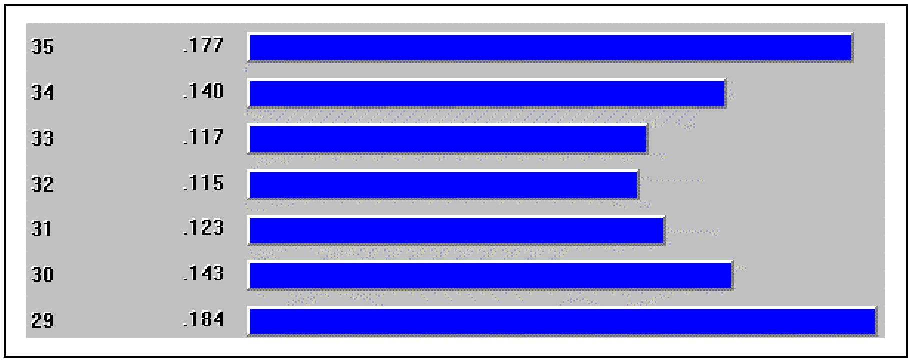


Figure 34 – Illustrative Resulting Priorities

the time, cost, labor stress and the likelihood of slippage can make use of the CPM analysis results in supporting management's decision-making.

Forecasting

AHP is useful in synthesizing information in order to make better decisions under conditions of uncertainty. Illustrations for a wide range of forecasting applications begin on page 213. These applications include:

- Synthesizing Forecasting Methods for a Composite forecast
- Selecting a Forecasting Method
- Deriving Probability Distributions
- Forecasting Alternative Outcomes
- Forecasting models interacting with choice model(s)
- Deriving Scenario Likelihoods
- Analytical Planning

Integer Programming

Suppose that we must decide on the appropriate combination of products to produce, subject to certain restrictions, such as budget limitations, diversification constraints, and dependency constraints. If we try to investigate each possible combination of products, two difficulties arise. First, how do we estimate the "overall" worth to the firm of a product or a specific combination of products? And second, if there is a relatively large number of products, the number of combinations is extremely large. For example, if we had 20 products and 10 constraints, we would have to consider more than 30 million combinations![31]

The problem can be formulated as an integer linear programming problem by defining decision variables X_i, i=1 to n, corresponding to the n products under consideration, where X_i will be equal to one if the ith product is to be produced, and zero if it is not. If we had a measure of the overall "worth" of each product to the firm, say W_i for the ith product, then we would like to maximize the sum of the worth over all products that will be included in the company's portfolio. This can be expressed as:

Maximize W1X1+W2X2+....WnXn (the worth of the products to be produced)

Subject to:
Budgetary constraint:
C1X1+C2X2+....CnXn <= Available budget

Diversification constraints:
(i.e. at least one product in each market segment, and
no more than two products in each market segment

Dependency constraints:
(e.g. either both products 1 and 2 or neither)

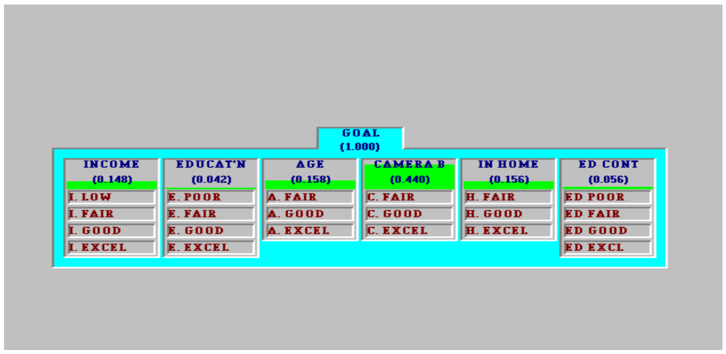
and Xi= 0 or 1.

The remaining difficulty, that of evaluating the worth (W_i) of each of the products can be solved using AHP. This approach allows one to consider all relevant considerations in the process of determining the "best" combination of products to produce.

---

[31]Examining only the extreme points of the convex hull would require (m+n)!/(m! x n!) or 30!/(20! x 10!)points to be examined.)

Evaluating Magazine Effectiveness



Abbreviation	Definition
A. EXCEL	MAGAZINE WITH > 60% READERS IN 18-44 AGE BRACKET IS EXCELLENT
A. FAIR	MAGAZINE WITH 40-50% READERS IN 18-44 AGE BRACKET IS FAIR
A. GOOD	MAGAZINE WITH 50-60% READERS IN 18-44 AGE BRACKET IS GOOD
AGE	% READERSHIP BETWEEN THE AGES OF 18 AND 44 YEARS OLD
C. EXCEL	MAGAZINE WITH >20% READERS PURCHASING CAMERA <1 YR IS EXCELLENT
C. FAIR	MAGAZINE WITH 10%-15% READERS PURCHASING CAMERA <1 YR AGO IS FAIR
C. GOOD	MAGAZINE WITH 15-20% READERS PURCHASING CAMERA <1 YR IS GOOD
CAMERA B	% READERSHIP THAT WERE CAMERA BUYERS WITHIN LAST 12 MONTHS
E. EXCEL	MAGAZINE WITH >60% READERS HAVING ANY COLLEGE IS EXCELLENT
E. FAIR	MAGAZINE WITH 40-50% OF READERS HAVING ANY COLLEGE IS FAIR
E. GOOD	MAGAZINE WITH 50-60% OF READERS HAVING ANY COLLEGE IS GOOD
E. POOR	MAGAZINE WITH UNDER 40% READERS HAVING ANY COLLEGE IS POOR
ED CONT	EDITORIAL CONTENT
ED EXCL	EXCELLENT EDITORIAL CONTENT MATCH
ED FAIR	FAIR EDITORIAL CONTENT MATCH
ED GOOD	GOOD EDITORIAL CONTENT MATCH
ED POOR	POOR EDITORIAL CONTENT MATCH
EDUCAT'N	% READERSHIP WITH SOME COLLEGE EDUCATION
H. EXCEL	MAGAZINE WITH >60% OF HOME READERS IS EXCELLENT
H. FAIR	MAGAZINE WITH 40-50% OF HOME READERS IS FAIR
H. GOOD	MAGAZINE WITH 50-60% OF HOME READERS IS GOOD
I. EXCEL	MAGAZINE WITH >20% OF READERS HAVING INCOME >$35,000 IS EXCELLENT

Figure 35 – Evaluating Magazine Effectiveness

Similar decisions to choosing a portfolio of products are the decision of which R&D projects to fund, and the decision of which magazines should be used for a marketing campaign.

Let us consider the choice of magazines for an advertising campaign for a 35 mm camera. Using an AHP model with the ratings approach, we can develop measures of effectiveness for each magazine with respect to objective criteria, such as income and age demographics of the readers of the magazines, as well as subjective criteria, such as editorial content.

Table 9 – Magazine Ratings and Total Effectiveness

ALTERNATIVES	INCOME .1482	EDUCAT'N .0425	AGE .1577	CAMERA B .4404	IN HOME .1557	ED CONT .0557	TOTAL
1 NATL GEOGRAPHIC	I. GOOD	E. GOOD	A. GOOD	C. GOOD	H. EXCEL	ED GOOD	0.311
2 NEWSWEEK	I. GOOD	E. GOOD	A. EXCEL	C. GOOD	H. FAIR	ED EXCL	0.331
3 SOUTHERN LIVING	I. LOW	E. FAIR	A. FAIR	C. GOOD	H. EXCEL	ED POOR	0.243
4 PEOPLE	I. LOW	E. FAIR	A. EXCEL	C. GOOD	H. FAIR	ED POOR	0.260
5 SPORTS ILLUS.	I. GOOD	E. FAIR	A. EXCEL	C. GOOD	H. GOOD	ED EXCL	0.346
6 TRAVEL & LEISURE	I. EXCEL	E. EXCEL	A. FAIR	C. EXCEL	H. FAIR	ED POOR	0.388
7 TIME	I. FAIR	E. GOOD	A. EXCEL	C. GOOD	H. GOOD	ED EXCL	0.327
8 U.S. NEWS	I. GOOD	E. GOOD	A. GOOD	C. EXCEL	H. FAIR	ED EXCL	0.402

Figure 35 illustrates an Expert Choice model used to derive such measures of effectiveness.

The pairwise comparison process yields priorities for the ratings as shown above. The global priorities are used when rating the magazines. Each magazine is given a rating with respect to each criterion. For example, with respect to the Income criterion, a magazine rated as Excellent would have .073 added to its effectiveness index, while a magazine rated Low would receive a value of .008. The ratings and total effectiveness for each magazine is shown in Table 9.

Next we must consider which combination of alternatives is “best”, subject to constraints. Suppose our only constraint is budget. If we know the budgetary requirements of each of the alternatives, we can formulate an integer linear programming model as shown in Table 11.

Table 10 – Integer Programming Solution

```
X1 (NATL GEOGRAPHIC) = 1
X2 (NEWSWEEK)        = 0
X3 (SOUTHERN LIVING) = 1
X4 (PEOPLE)          = 1
X5 (SPORTS ILLUS.)   = 0
X6 (TRAVEL & LEISURE)= 1
X7 (TIME)            = 0
X8 (U.S. NEWS)       = 1
```

The final integer programming (IP) solution, shown in Table 10, concludes that advertisements should be placed in National Geographic, Southern Living, People, Travel & Leisure, and U.S. News magazines.

The "optimal" solution from the ILP formulation should not be taken as the final decision. Rather, it must be examined to see if it suggests other criteria that should be added to the AHP formulation, and/or a re-evaluation of judgments in the AHP model, and/or additional constraints for the ILP model. Iteration is performed until an acceptable, "optimal" solution is achieved.

Table 11 – Integer Linear Programming Formulation

```
Maximize  E1 R1 X1  + E2 R2 X2  +   .... + E8 R8 X8,
   where
    Magazine 1 is National Geographic,
    R1 = 21,051 (the number of readers)
    E1 = .311 (the effectiveness coefficient from the Ratings model),
    X1 will be determined and will be 1 if it is optimal to
       advertise in National Geographic, 0 otherwise,

    Magazine 2 is Newsweek
    R2 = 15,594 (the number of readers)
    E2 = .331 (the effectiveness coefficient from the Ratings model),
    X2 will be determined and will be 1 if it is optimal to
       advertise in Newsweek, 0 otherwise,
  .
  .
  .

    Magazine 8 is U.S. News
    R8 =  8,929 (the number of readers)
    E8 = .402 (the effectiveness coefficient from the Ratings model),
    X8 will be determined and will be 1 if it is optimal to
       advertise in U. S. News, 0 otherwise.

Subject to the constraint on the total advertising budget:

     346,080 X1 + 780,180 X2 + 11,370 X3 + 605,880 X4 + 965,940 X5 +
                  183,216 X6 + 1,324,282 X7 + 100,740 X8 <= $1,500,000
```

The same approach can be used to determine the best combination of R&D projects for a company. Objectives or criteria such as market position, fit with strategic direction, and projected sales can be used in the AHP model. Constraints that preclude too much redundancy or require a minimum amount of research in a given area can easily be included in the ILP model.

Care must be taken to assure that the decision truly reflects management's objectives and constraints. Not only can a piecemeal analysis be difficult to synthesize into the decision process, but the results prove to be troublesome. As an example, a Fortune 500 company used AHP to rate R&D projects. They were satisfied with both the process of arriving at the priorities and the priorities themselves. However, they did not think through the resource allocation problem thoroughly and simply allocated funds from their budget to the projects in rank order until no more funding remained. This resulted in some obvious weaknesses. Some departments got very large increases in funding while others got very large decreases. The departments with large increases were happy and quiet. The departments with large decreases were unhappy and very vocal. Something was wrong with the process! Furthermore, it appeared that some research areas had an overabundance of funding while others had too little funding. With a little bit more thought about objectives and constraints, the resource allocation could have been greatly improved. One objective of the organization was to keep their employee morale high. Employee morale in those departments with large reductions in funding suffered.

Management could have included constraints in the ILP formulation that guaranteed a somewhat smoother transition from the present R&D funding to a more desirable one. For example, constraints that guaranteed that each department get at least a given percentage of the previous years allocation would have prevented any drastic changes that adversely effected employee morale. Other constraints to guarantee a minimum amount of diversification and a minimum amount of coverage to specific research areas could have easily been accommodated. Thus, with a little thought about the objectives and constraints, and with some iteration, the AHP/ILP combination is a powerful mechanism for allocating resources so as to "best" meet an organization's objectives. A more detailed discussion of resource allocation begins on page 235.